# Universidad Carlos III de Madrid 

Escuela Politécnica Superior
Departamento de Matemáticas

# DIFFERENTIAL EQUATIONS <br> SELFEVALUATION I 

24th to 31st of October, 2017
Degree in Biomedical Engineering
Time: 1 hour
The marking is only valid to check the learning pace. It does not compute for the final note.
Problem 1 (2 points)
Solve the equation: $x y^{\prime} \sin \frac{y}{x}=x+y \sin \frac{y}{x}$.

## Problem 2 (2 points)

Solve, using an integrating factor that depends only on one variable:

$$
y^{2} \cos (x) d x+(4+5 y \sin (x)) d y=0
$$

## Problem 3 (2 points)

Solve by reduction of order: $y y^{\prime \prime}+\left(y^{\prime}\right)^{2}=0$.

## Problem 4 (2 points)

Solve the equation: $x^{3} y^{\prime \prime \prime}+x y^{\prime}-y=3 x^{4}$.

Problem 5 (2 points)
Use Laplace transform to solve for $\omega \neq \omega_{0}$ the initial value problem

$$
\left\{\begin{array}{l}
x^{\prime \prime}+\omega_{0}^{2} x=k \sin \omega t, \quad t>0 \\
x(0)=x^{\prime}(0)=0
\end{array}\right.
$$

which describes the forced oscillations of a mass on an undamped spring. What happens if $\omega=\omega_{0}$ ?
Hint: Verify that $L^{-1}\left\{\frac{1}{\left(s^{2}+a^{2}\right)^{2}}\right\}=\frac{1}{2 a^{3}}(\sin a t-a t \cos a t)$.

