Universidad Carlos III de Madrid Escuela Politécnica Superior

Departamento de Matemáticas

DIFFERENTIAL EQUATIONS SELFEVALUATION I

24th to 31st of October, 2017 Degree in Biomedical Engineering

Time: 1 hour

The marking is only valid to check the learning pace. It does not compute for the final note.

Problem 1 (2 points)

Solve the equation: $xy' \sin \frac{y}{x} = x + y \sin \frac{y}{x}$.

Problem 2 (2 points)

Solve, using an integrating factor that depends only on one variable:

$$y^{2}\cos(x)dx + (4+5y\sin(x))dy = 0.$$

Problem 3 (2 points)

Solve by reduction of order: $yy'' + (y')^2 = 0$.

Problem 4 (2 points)

Solve the equation: $x^3y''' + xy' - y = 3x^4$.

Problem 5 (2 points)

Use Laplace transform to solve for $\omega \neq \omega_0$ the initial value problem

$$\begin{cases} x'' + \omega_0^2 x = k \sin \omega t, \quad t > 0\\ x(0) = x'(0) = 0 \end{cases}$$

which describes the forced oscillations of a mass on an undamped spring. What happens if $\omega = \omega_0$?

Hint: Verify that $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\} = \frac{1}{2a^3}(\sin at - at\cos at).$