

DIFFERENTIAL EQUATIONS
SELF-EVALUATION II
5th to 12th of December, 2017
Degree in Biomedical Engineering.

Time: 90 minutes

The marking is only valid to check the learning pace. It does not compute for the final note.

Problem 1 (4 points)

Solve the Laplace equation over a semicircle:

$$\Delta u = 0, \quad \{0 < r < a, 0 < \theta < \pi\}$$

with the boundary conditions:

The diameter is isolated and: $u(a, \theta) = g(\theta)$.

Problem 2 (2 points)

Use the Rayleigh quotient to obtain a reasonably precise upper bound for the first eigenvalue of the problem:

$$\varphi'' + (\lambda - x)\varphi = 0, \quad \varphi'(0) = 2\varphi(1) + \varphi'(1) = 0.$$

Problem 3 (4 points)

Solve the telegraph problem:

$$\begin{cases} u_{tt} - u_{xx} + au_t + bu = 0 & 0 < x < L, t > 0, \\ u(0, t) = u(L, t) = 0 & t > 0, \\ u(x, 0) = f(x) & 0 < x < L, \\ u_t(x, 0) = 0 & 0 < x < L. \end{cases}$$
