# Universidad Carlos III de Madrid
**Escuela Politécnica Superior**
Departamento de Matemáticas

## DIFFERENTIAL EQUATIONS
### CONTROL 2
18th of December 2017
Degrees in Biomedical Engineering.

**Time: 3 hours**

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**Problem 1  (4 points)**
Solve the following problem on a rectangle:

$$\begin{cases} u_{xx} + 4u_{yy} = 0, & 0 < x < \pi/2, \quad 0 < y < \pi, \\ u(0,y) = u(\pi/2, y) = 0, & 0 < y < \pi, \\ u(x, \pi) = 0, & 0 < x < \pi/2, \\ u(x, 0) = 2\sin 6x, & 0 < x < \pi/2 \,. \end{cases}$$

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**Problem 2  (1 + 2 points)**
Consider the Laplace problem in a disc with radius 3:

$$\begin{cases} \Delta u = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2} = 0, & 0 < r < 3, \\[2mm] u(3, \theta) = f(\theta). \end{cases}$$

**a)** Apply the method of separate variables and find the one variable problems.

**b)** Solve those problems (ODE's).

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**Problem 3  (1,5 + 1,5 points)**
Consider the problem:

$$\begin{cases} \varphi'' + 2\varphi' + (\lambda - x)\varphi = 0, & 0 < x < 1, \\ \varphi'(0) = \varphi(1) = 0. \end{cases}$$

**a)** Write it in form of a Sturm-Liuville problem using an integrating factor.

**b)** Study if all the eigenvalues are positive and if there is a zero eigenvalue.

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