

**DIFFERENTIAL EQUATIONS  
EXTRAORDINARY EXAM**

18th of June, 2018  
Degree in Biomedical Engineering.

**Time: 3 hours**

---

**Problem 1 (1.5 points)**

Solve the equation  $3xy^2y' + y^3 = x \sin x$ .

---

**Problem 2 (2 points)**

Solve the equation  $xy'' + 3y' + \frac{1}{x}y = x^2$ .

---

**Problem 3 (2 points)**

Solve the initial value problem

$$\begin{cases} y'' + 2y' + 2y = g(t), \\ y(0) = 1, \quad y'(0) = 0, \end{cases} \quad g(t) = \begin{cases} 3, & 0 < t < 2\pi, \\ 0, & 2\pi \leq t < 5\pi, \\ 1, & 5\pi \leq t. \end{cases}$$

---

**Problem 4 (0.5 + 1 + 1 = 2.5 points)**

a) Split into two one-variable problems

$$\begin{cases} u_t - 2u_{xx} = 2u, & -\frac{\pi}{2} < x < \frac{\pi}{2}, \quad t > 0, \\ u(-\pi/2, t) = u(\pi/2, t), & t > 0, \\ u_x(-\pi/2, t) = u_x(\pi/2, t), & t > 0, \\ u(x, 0) = 8 - 3\sin(2x) - 8\cos^2(2x), & -\frac{\pi}{2} < x < \frac{\pi}{2}. \end{cases}$$

b) Solve both problems.

c) Find the solution of the original problem.

---

**Problem 5 (0,5 + 0,5 + 1 = 2 points)**

a) Prove that the following problem is not of Sturm-Liouville type and transform it into one:

$$\begin{cases} x^2\phi'' + x\phi' + \lambda\phi = 0, \\ \phi(1) = 0, \quad \phi(e) = 0. \end{cases}$$

b) Prove that all the eigenvalues are positive.

c) Find the eigenvalues and eigenfunctions of the problem.

---