

DIFFERENTIAL EQUATIONS  
FINAL EXAM

22th of January, 2018  
Degree in Biomedical Engineering.

Time: 3 hours

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**Problem 1 (1.5 points)**

Solve the equation

$$\frac{dy}{dx} - 5y = -\frac{5}{2}xy^3.$$

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**Problem 2 (2 points)**

Find the general solution of the equation:

$$y'' + y = \operatorname{tg}(x).$$

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**Problem 3 (1.5 points)**

Solve the integro-differential equation

$$f'(x) + \int_0^x 4 \cdot f(x-t) dt = x - \sin x, \quad f(0) = 2.$$

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**Problem 4 (2.5 points)**

Solve, using separation of variables, the telegraph equation

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} + 2 \frac{\partial u}{\partial t} + u = \frac{\partial^2 u}{\partial x^2}, & t > 0, \quad x \in (0, \pi), \\ u(0, t) = u(\pi, t) = 0, & t > 0, \\ u(x, 0) = 4 \sin(2x), & x \in [0, \pi], \\ \frac{\partial u}{\partial t}(x, 0) = 0, & x \in [0, \pi]. \end{cases}$$

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**Problem 5 (0,5 + 0,5 + 1,5 = 2.5 points)**

a) Transform the following into a Sturm-Liouville problem:

$$\begin{cases} r \frac{d}{dr} \left( r \frac{d\phi}{dr} \right) + \lambda r^2 \phi = 0, & 0 < r < 2, \\ \phi(2) = 0, \\ |\phi(0)| < \infty. \end{cases}$$

b) Prove that all the eigenvalues are positive and determine the orthogonality relation satisfied by the eigenfunctions.

c) Find the eigenfunctions and eigenvalues.

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