Module 4: Analysis of three-phase systems

Electrical power engineering fundamentals

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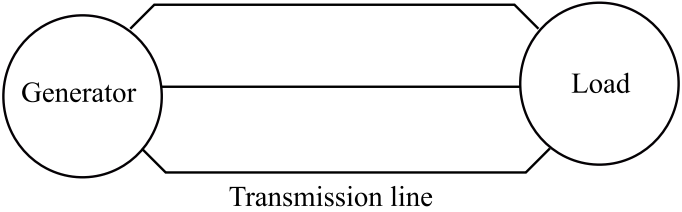
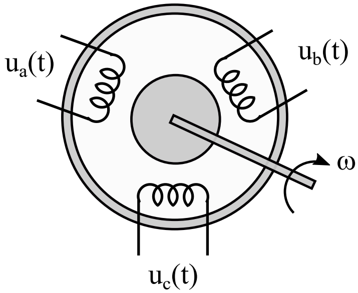
Configuration of three-phase systems

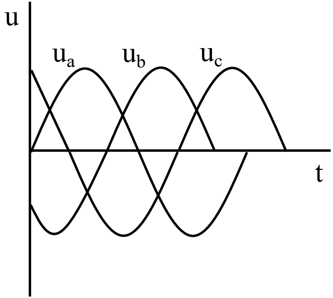
- Most power systems are three-phase.
- Each part of the system is called **phase**



Three phase generators

Three phase generators generate three sinusoidal voltages of the same amplitude and phase shift 120^o





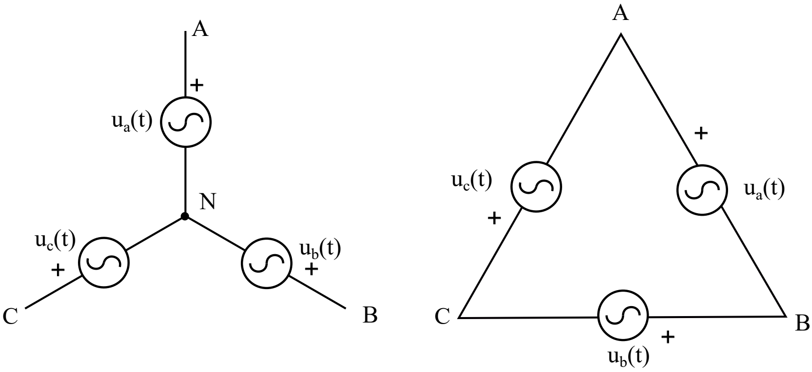
$$u_a(t) = \sqrt{2} \cdot U \cdot \cos(\omega t)$$

$$u_b(t) = \sqrt{2} \cdot U \cdot \cos(\omega t - 120^o)$$

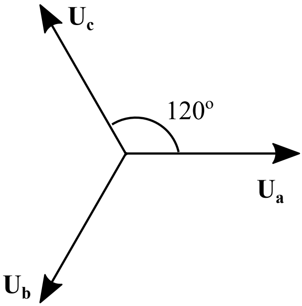
$$u_c(t) = \sqrt{2} \cdot U \cdot \cos(\omega t + 120^o)$$

Generators in wye and delta

We represent three-phase generators as three AC sources connected in **wye** or **delta**

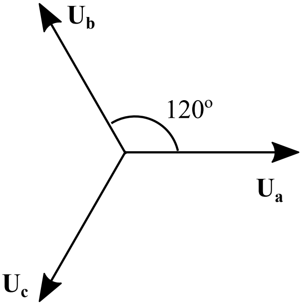


Positive and negative phase sequence



Positive sequence

$$\underline{\mathbf{U}_a} = U\angle 0^o$$
$$\underline{\mathbf{U}_b} = U\angle -120^o$$
$$\underline{\mathbf{U}_c} = U\angle 120^o$$

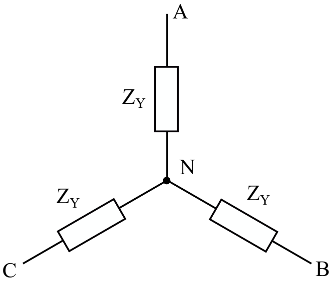


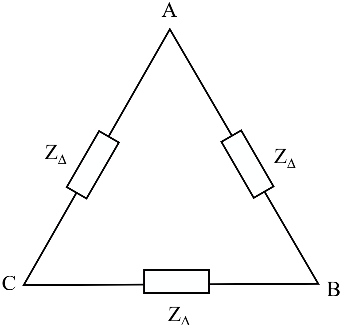
Negative sequence

$$\underline{\mathbf{U}_a} = U\angle 0^o$$
$$\underline{\mathbf{U}_b} = U\angle 120^o$$
$$\underline{\mathbf{U}_c} = U\angle -120^o$$

Three-phase loads

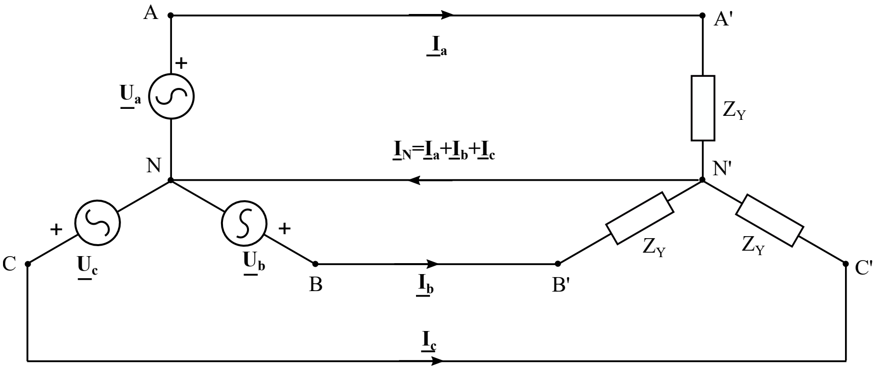
- Three-phase loads are represented in the frequency domain, as a set of three impedances connected between them.
- We limit our analysis to **balanced systems** in which the impedance connected to each phase has the same value
- Loads can be connected in wye or delta





Balanced wye-wye system systems

In wye wye systems the neutral points of the generators and the loads might be connected by means of a **neutral wire**.

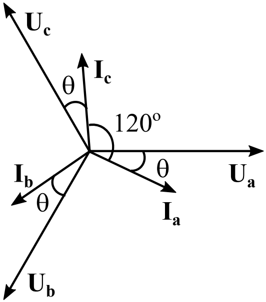


The current of each phase flows from the generator towards the load and returns through the neutral wire

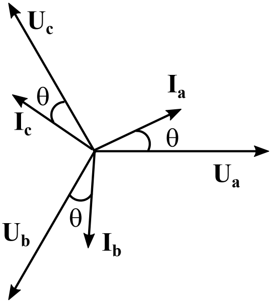
Currents in a balanced wye-wye system

$$\underline{\mathbf{I}}_a = \frac{\underline{\underline{\mathbf{U}}}_a}{Z_Y} = \frac{U\angle 0}{Z_Y} \qquad \underline{\mathbf{I}}_b = \frac{\underline{\underline{\mathbf{U}}}_b}{Z_Y} = \frac{U\angle -120^o}{Z_Y} \qquad \underline{\mathbf{I}}_c = \frac{\underline{\underline{\mathbf{U}}}_c}{Z_Y} = \frac{U\angle 120^o}{Z_Y}$$

The currents also form a three phase system. If $Z_Y = |Z_Y|\angle\theta$



Inductive load

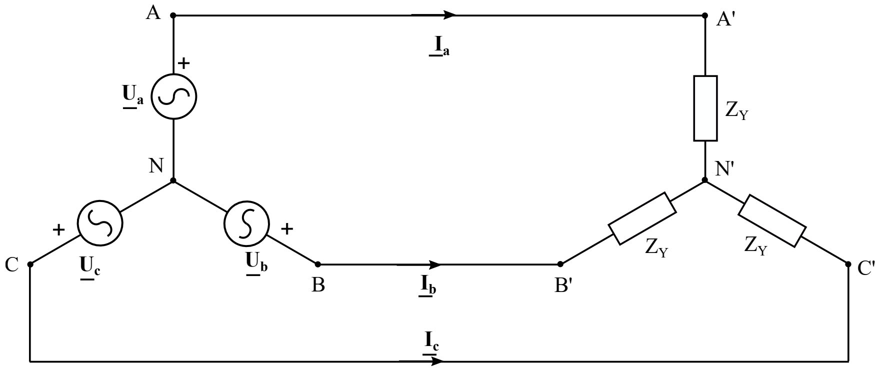


Capacitive load

Current through the neutral wire

As the current flowing through the neutral is zero the wire is often suppressed.

$$\underline{\mathbf{I}_N} = \underline{\mathbf{I}_a} + \underline{\mathbf{I}_b} + \underline{\mathbf{I}_c} = \frac{U}{Z_Y} \cdot (1\angle 0 + 1\angle - 120^o + 1\angle 120^o) = 0$$



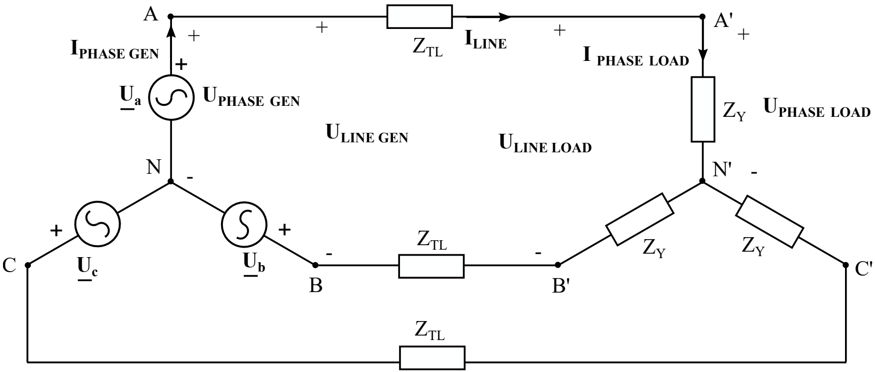
Savings of three-phase system

- Lower material investment
- Lower line losses for the same power transfer

Line and phase voltages and currents

- **Phase-voltage**: Voltage drop across a single phase of the generator or the load. The phase-voltage of the generator is the voltage drop across the terminals of one of the ideal voltage sources; the phase voltage of the load is the voltage drop across one of the impedances that constitutes the load.
- **Line-votage**: Voltage drop between any couple of lines. We could obtain the line voltage at the generator side of the system, or the line voltage at the load side.
- **Phase current**: Current in a single phase, i.e current flowing through one of the ideal sources or through one of the impedances.
- **Line current**: Current in a single line

Line and phase voltages and currents



Relation between the line and phase magnitudes in a wye-wye system

Relation between the line and phase voltages:

$$\underline{\mathbf{I}}_L = \underline{\mathbf{I}}_{Ph}$$

Relation between the line and phase voltages:

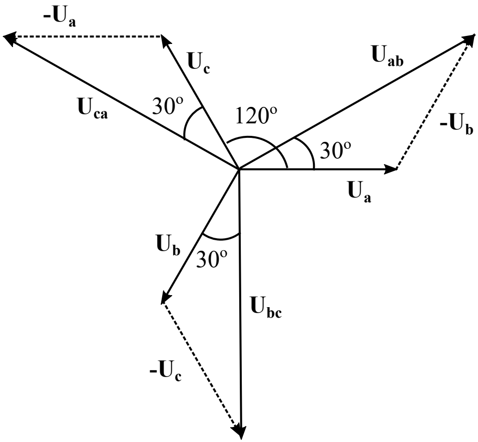
$$\underline{\mathbf{U}}_{Pha} = \underline{\mathbf{U}}_a \qquad \underline{\mathbf{U}}_{Phb} = \underline{\mathbf{U}}_b \qquad \underline{\mathbf{U}}_{Phc} = \underline{\mathbf{U}}_c$$

$$\underline{\mathbf{U}}_{La} = \underline{\mathbf{U}}_a - \underline{\mathbf{U}}_b = U\angle 0 - U\angle -120^o = \sqrt{3}\cdot U\angle 30^o = \sqrt{3}\cdot \underline{\mathbf{U}}_{Pha}\angle 30^o$$

$$\underline{\mathbf{U}}_{Lb} = \underline{\mathbf{U}}_{BC} = \underline{\mathbf{U}}_b - \underline{\mathbf{U}}_c = \sqrt{3}\cdot \underline{\mathbf{U}}_{Phb}\angle 30^o$$

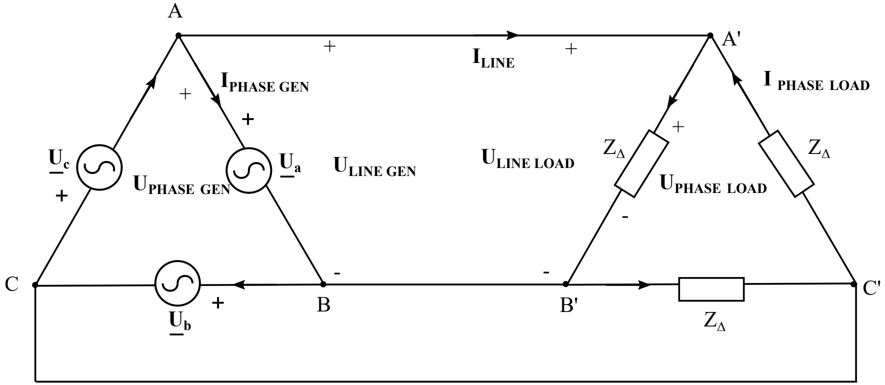
$$\underline{\mathbf{U}}_{Lc} = \underline{\mathbf{U}}_{CA} = \underline{\mathbf{U}}_c - \underline{\mathbf{U}}_a = \sqrt{3}\cdot \underline{\mathbf{U}}_{Phc}\angle 30^o$$

Relation between the line and phase magnitudes in a wye-wye system



The line voltages are $\sqrt{3}$ times larger that the phase voltages and lead the phase voltages by 30^o

Relation between the line and phase magnitudes in a delta-delta system



Relation between the line and phase magnitudes in a delta-delta system

Phase and line voltages:

$$\underline{\mathbf{U}}_{LINE} = \underline{\mathbf{U}}_{PHASE}$$

Phase currents:

$$\underline{\mathbf{I}}_{Pha} = \underline{\mathbf{I}}_{B'C'} = \frac{\underline{\mathbf{U}}_a}{Z_\Delta} = \frac{U\angle 0}{Z_\Delta}$$

$$\underline{\mathbf{I}}_{Phb} = \underline{\mathbf{I}}_{A'B'} = \frac{\underline{\mathbf{U}}_b}{Z_\Delta} = \frac{U\angle -120^o}{Z_\Delta}$$

$$\underline{\mathbf{I}}_{Phc} = \underline{\mathbf{I}}_{C'A'} = \frac{\underline{\mathbf{U}}_c}{Z_\Delta} = \frac{U\angle 120^o}{Z_\Delta}$$

Relation between the line and phase magnitudes in a delta-delta system

Line currents:

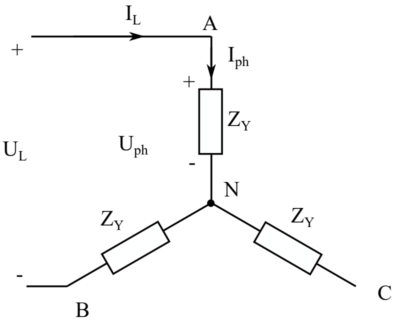
$$\underline{\mathbf{I}}_{La} = \underline{\mathbf{I}}_{Pha} - \underline{\mathbf{I}}_{Phb} = \frac{U\angle 0}{Z_{\Delta}} - \frac{U\angle -120^{o}}{Z_{\Delta}} = \sqrt{3} \cdot \frac{U}{Z_{\Delta}} \angle -30^{o} = \sqrt{3} \cdot \underline{\mathbf{I}}_{Pha} \angle -30^{o}$$

$$\underline{\mathbf{I}}_{Lb} = \sqrt{3} \cdot \underline{\mathbf{I}}_{Phb} \angle -30^{o}$$

$$\underline{\mathbf{I}}_{Lc} = \sqrt{3} \cdot \underline{\mathbf{I}}_{Phc} \angle -30^{o}$$

The line currents are $\sqrt{3}$ **times larger that the phase currents and lag the phase currents by** 30^{o}

Summary



Wye configuration

$$U_{line} = \sqrt{3} \cdot U_{phase}$$

$$I_{line} = I_{phase}$$

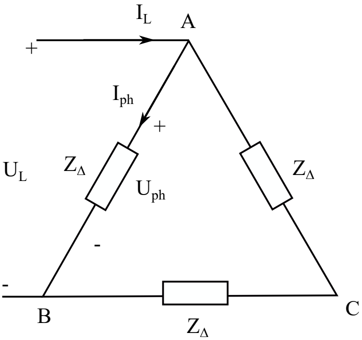
$$\underline{\mathbf{U}}_{LA} = \sqrt{3} \cdot \underline{\mathbf{U}}_{phA} \angle 30$$

Delta configuration

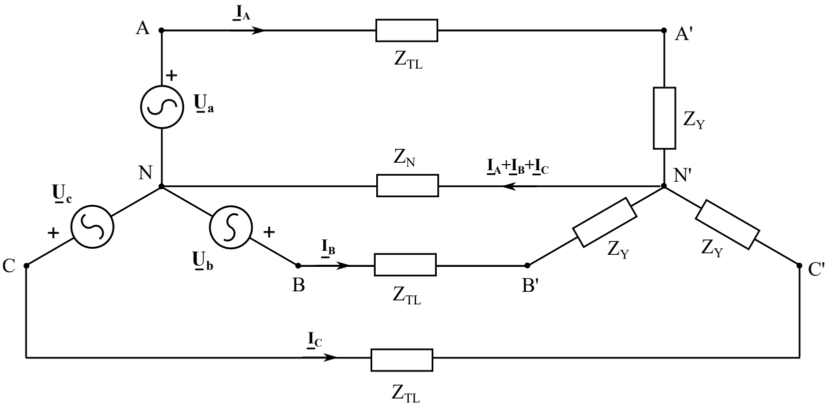
$$U_{line} = U_{phase}$$

$$I_{line} = \sqrt{3} \cdot I_{phase}$$

$$\underline{\mathbf{I}}_{LA} = \sqrt{3} \cdot \underline{\mathbf{I}}_{phA} \angle -30$$



Analysis of three-phase systems: one-phase equivalent

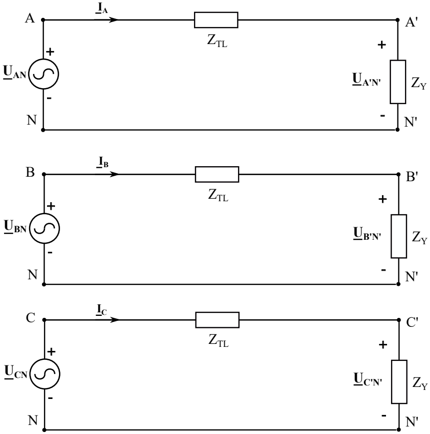


$$-\underline{\mathbf{U}}_a + \underline{\mathbf{I}}_a \cdot (Z_{TL} + Z_Y) + (\underline{\mathbf{I}}_a + \underline{\mathbf{I}}_b + \underline{\mathbf{I}}_c) \cdot Z_N = 0$$

$$-\underline{\mathbf{U}}_b + \underline{\mathbf{I}}_b \cdot (Z_{TL} + Z_Y) + (\underline{\mathbf{I}}_a + \underline{\mathbf{I}}_b + \underline{\mathbf{I}}_c) \cdot Z_N = 0$$

$$-\underline{\mathbf{U}}_c + \underline{\mathbf{I}}_c \cdot (Z_{TL} + Z_Y) + (\underline{\mathbf{I}}_a + \underline{\mathbf{I}}_b + \underline{\mathbf{I}}_c) \cdot Z_N = 0$$

Analysis of three-phase circuits



As there is no current flow through the neutral wire, the three phases can be analysed as independent circuits

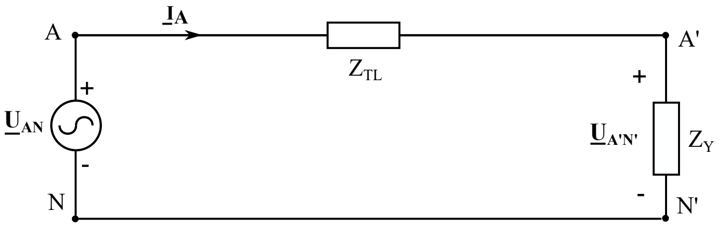
$$-\underline{\mathbf{U}}_a + \underline{\mathbf{I}}_a \cdot (Z_{TL} + Z_Y) = 0$$

$$-\underline{\mathbf{U}}_b + \underline{\mathbf{I}}_b \cdot (Z_{TL} + Z_Y) = 0$$

$$-\underline{\mathbf{U}}_c + \underline{\mathbf{I}}_c \cdot (Z_{TL} + Z_Y) = 0$$

One phase equivalent of a three-phase circuit

- We represent the circuit with a one-phase equivalent.
- As the system is balanced the electric magnitudes of the three phases have the same amplitude and a known phase shift (120^o).
- The behaviour of the whole system could be derived from the analysis of the so called **one-phase equivalent** or **phase-neutral equivalent** of the system.

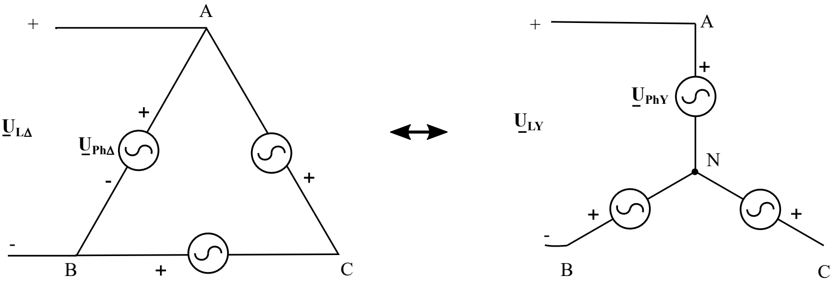


Analysis of circuits with delta-connected elements

- In systems with a delta-connected load or a delta-connected generator, it is not possible to apply the one-phase equivalent approach directly, since there is no neutral point.
- However it is possible to apply a wye delta transformation to obtain a YY connected system equivalent to the original.
- In YΔ or ΔΔ or ΔY systems:
    1. The system is transformed into a YY equivalent system.
    2. The one-phase equivalent approach is applied over the YY equivalent circuit.

ΔY transformation for delta-connected generators

Δ connected generators can be redrawn as equivalent Y connected generators:

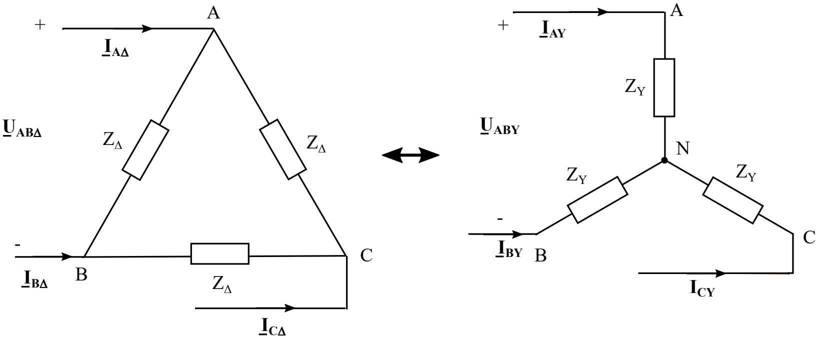


The two configurations are equivalent if:

$$\underline{\mathbf{U}}_{LY} = \underline{\mathbf{U}}_{L\Delta}$$

ΔY transformation for three delta-connected impedances

We want to find the value of Z_Y that makes the two loads equivalent



The loads are equivalent if for the same applied line voltages the line currents are the same (i.e: $\underline{\mathbf{U}}_{AB\Delta} = \underline{\mathbf{U}}_{ABY} => \underline{\mathbf{I}}_{A\Delta} = \underline{\mathbf{I}}_{AY}$)

ΔY transformation for three delta-connected impedances

The behaviour in the three phases is the same except that there is a phase shift of 120^o. We analyse phase A:

$$\underline{\mathbf{I}}_{A\Delta} = \sqrt{3} \cdot \underline{\mathbf{I}}_{PhA} \angle -30^o = \frac{\underline{\mathbf{U}}_{AB} \cdot \sqrt{3} \angle -30^o}{Z_\Delta}$$

$$\underline{\mathbf{I}}_{AY} = \underline{\mathbf{I}}_{PhA} = \frac{\underline{\mathbf{U}}_{AB} / \sqrt{3} \angle 30^o}{Z_Y}$$

What value of impedance Z_Y verifies the identity $\underline{\mathbf{I}}_{A\Delta} = \underline{\mathbf{I}}_{AY}$?:

$$\frac{\underline{\mathbf{U}}_{AB} \cdot \sqrt{3} \angle -30^o}{Z_\Delta} = \frac{\underline{\mathbf{U}}_{AB} / \sqrt{3} \angle 30^o}{Z_Y} \quad => \quad \boxed{Z_Y = \frac{Z_\Delta}{3}}$$

Instantaneous power

The instantaneous power of three phase systems is constant*:

$$p(t) = u_a(t) \cdot i_a(t) + u_b(t) \cdot i_b(t) + u_c(t) \cdot i_c(t) = 3 \cdot U \cdot I \cdot \cos\varphi$$

As power is constant the vibrations in the axles of three phase motors and generators are smaller that those in one-phase devices what makes them more stable from the mechanical point of view.

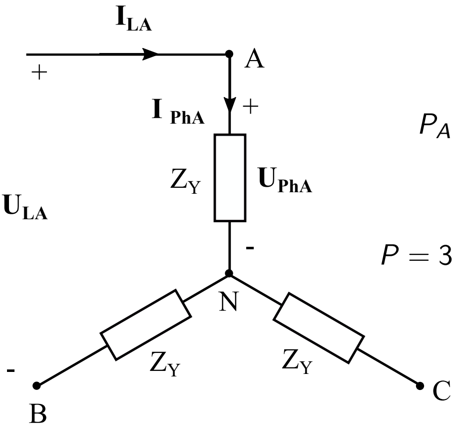
$$u_a(t) = \sqrt{2} \cdot U \cdot \cos(\omega t) \qquad i_a(t) = \sqrt{2} \cdot I \cdot \cos(\omega t - \varphi)$$

$$u_b(t) = \sqrt{2} \cdot U \cdot \cos(\omega t - 120^o) \qquad i_b(t) = \sqrt{2} \cdot I \cdot \cos(\omega t - 120^o - \varphi)$$

$$u_c(t) = \sqrt{2} \cdot U \cdot \cos(\omega t + 120^o) \qquad i_c(t) = \sqrt{2} \cdot I \cdot \cos(\omega t + 120^o - \varphi)$$

*Read the demonstration in the long notes

Active and reactive power of a three-phase wye load



$$P = P_A + P_B + P_C$$

$$P_A = P_B = P_C = U_{Ph} \cdot I_{Ph} \cdot \cos\varphi$$

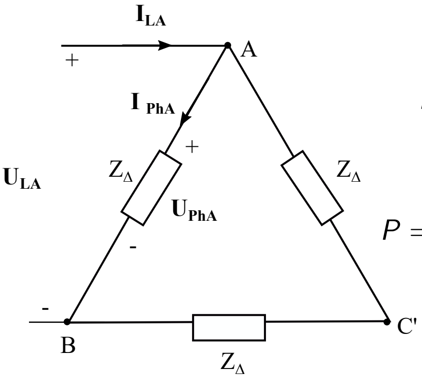
$$P = 3 \cdot U_{Ph} \cdot I_{Ph} \cdot \cos\varphi = \sqrt{3} \cdot U_L \cdot I_L \cdot \cos\varphi$$

$$I_L = I_{Ph} \qquad U_{Ph} = U_L / \sqrt{3}$$

$$Z_Y = |Z_Y| \angle \theta$$

$$Q = 3 \cdot U_{Ph} \cdot I_{Ph} \cdot \sin\varphi = \sqrt{3} \cdot U_L \cdot I_L \cdot \sin\varphi$$

Active and reactive power of a three-phase delta load



$$P = P_A + P_B + P_C$$

$$P_A = P_B = P_C = U_{Ph} \cdot I_{Ph} \cdot \cos\varphi$$

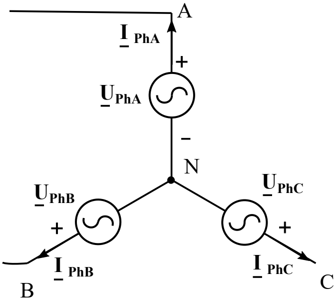
$$P = 3 \cdot U_{Ph} \cdot I_{Ph} \cdot \cos\varphi = \sqrt{3} \cdot U_L \cdot I_L \cdot \cos\varphi$$

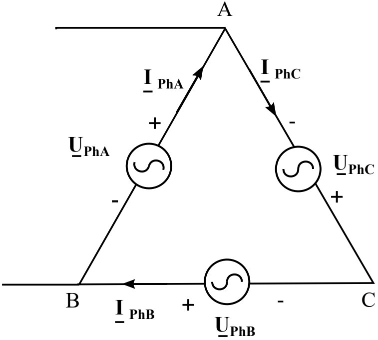
$$U_L = U_{Ph} \qquad I_{Ph} = I_L/\sqrt{3}$$

$$Z_\Delta = |Z_\Delta| \angle \theta$$

$$Q = 3 \cdot U_{Ph} \cdot I_{Ph} \cdot \sin\varphi = \sqrt{3} \cdot U_L \cdot I_L \cdot \sin\varphi$$

Complex power of three-phase generators

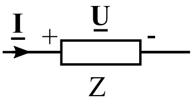


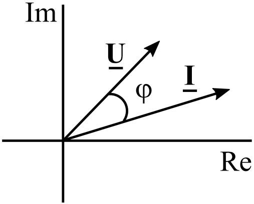


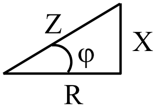
$$\mathcal{S}_g = \underline{\mathbf{U}}_{PhA} \cdot \underline{\mathbf{I}}^*_{PhA} + \underline{\mathbf{U}}_{PhB} \cdot \underline{\mathbf{I}}^*_{PhB} + \underline{\mathbf{U}}_{PhC} \cdot \underline{\mathbf{I}}^*_{PhC} = 3 \cdot \underline{\mathbf{U}}_{PhA} \cdot \underline{\mathbf{I}}^*_{PhA}$$

Power factor

Power factor: $\quad p.f. = \cos\varphi$





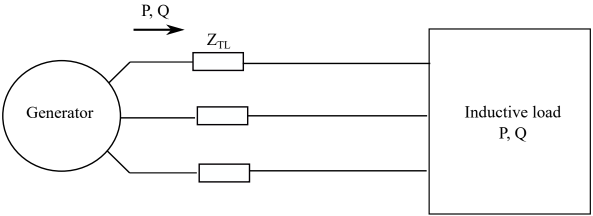


$$\varphi = \varphi_u - \varphi_i = \arctan\frac{Q}{P} = \arctan\frac{X}{R}$$

	Q	φ	$\cos\varphi$	**character**
Resistive loads	0	0	1	-
Inductive loads	> 0	> 0	$0 < p.f < 1$	lagging or inductive
Capacitive loads	< 0	< 0	$0 < p.f < 1$	leading or capacitive

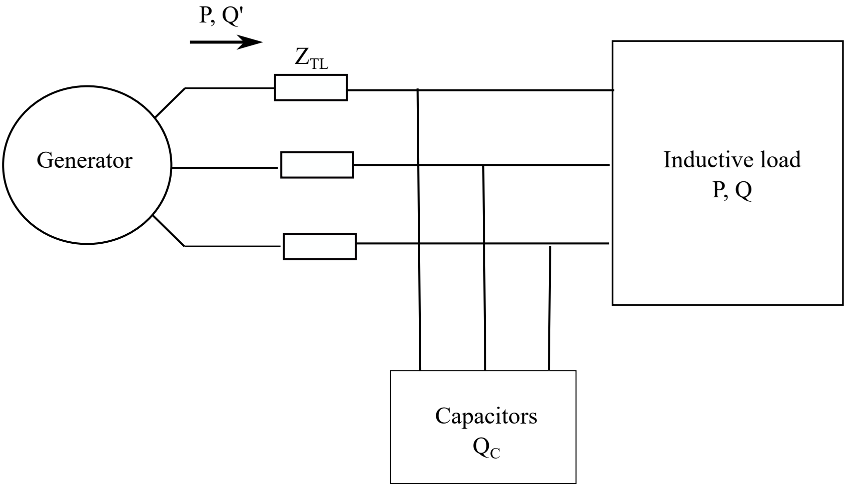
Reactive power compensation

- Many real-life loads, as electric motors, are highly inductive and often operation of power systems electric systems involves high amounts of reactive power transferred from the generators towards the loads.
- Fluctuating power increases the current flowing through the lines increasing losses and giving rise to voltage drops.
- Electric companies penalize the costumers that consume power with poor power factor.



Reactive power compensation

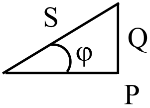
Banks of capacitors are connected in parallel with the loads, to compensate part of the reactive power absorbed by them.



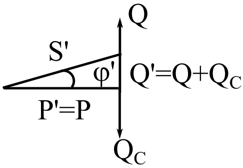
Reactive power compensation

- Capacitors do not absorb or deliver any active power, so the active power of the system remains unchanged. $Q_C < 0$
- The relation between the active an reactive power changes and the angle φ' becomes smaller.
- The power factor becomes closer to 1

Initial system



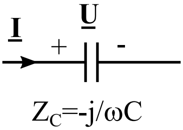
System with capacitors



$$Q' = Q + Q_C$$

Reactive power of a capacitor

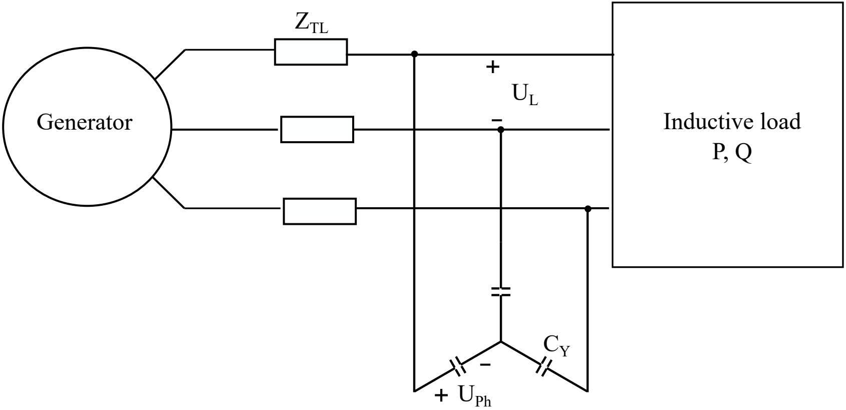
A capacitor of capacitance C, a voltage drop $\underline{\mathbf{U}} = U\angle\varphi_u$ and current flow $\underline{\mathbf{I}} = I\angle\varphi_i$



$$Q_C = X_C \cdot I^2 = \frac{U^2}{X_C} = -\omega \cdot C \cdot U^2$$

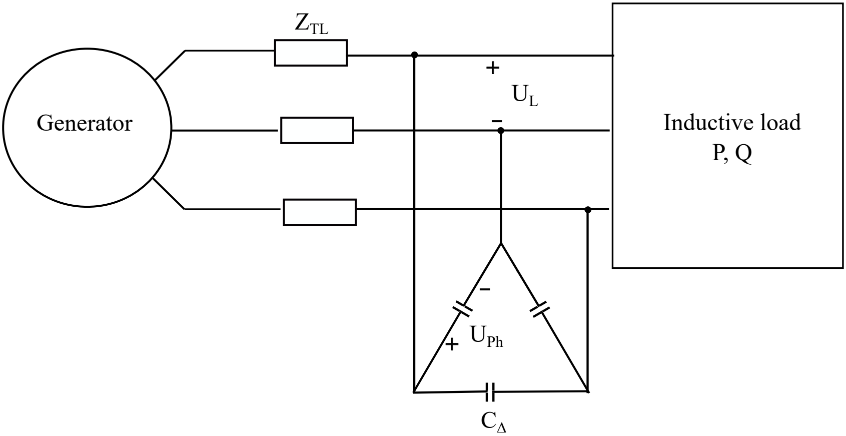
$$P_C = 0$$

Reactive power of a bank of capacitors in wye



$$Q_{CY} = -3 \cdot \omega \cdot C_Y \cdot \underbrace{U_{Ph}}_{U_L/\sqrt{3}}^{2} = -\omega \cdot C_Y \cdot U_L^2$$

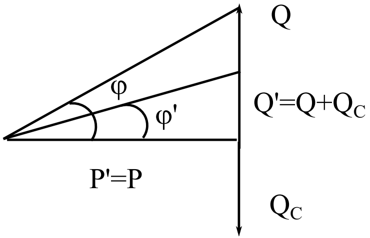
Reactive power of a bank of capacitors in delta



$$Q_{C\Delta} = -3 \cdot \omega \cdot C_\Delta \cdot \underbrace{U_{Ph}}_{U_L}{}^2 = -3 \cdot \omega \cdot C_\Delta \cdot U_L^2 \qquad (1)$$

Capacitance required to get a target power factor

We want to compensate the reactive power of a system working with power factor $\cos\varphi$ so that the power factor becomes $\cos\varphi'$



$$Q = P{\cdot}\tan\varphi \qquad Q' = P{\cdot}\tan\varphi' \qquad Q_C = Q - Q' = P{\cdot}(\tan\varphi - \tan\varphi')$$

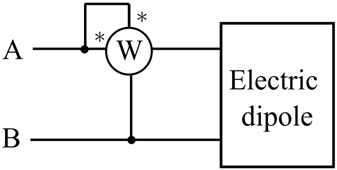
$$C_\Delta = \frac{P\cdot(\tan\varphi - \tan\varphi')}{3\cdot\omega\cdot U_L^2} \qquad C_Y = \frac{P\cdot(\tan\varphi - \tan\varphi')}{\omega\cdot U_L^2}$$

Measure of power: working principle of wattmeters

A wattmeter is a measuring device that provides information on the power absorbed by electric dipoles.

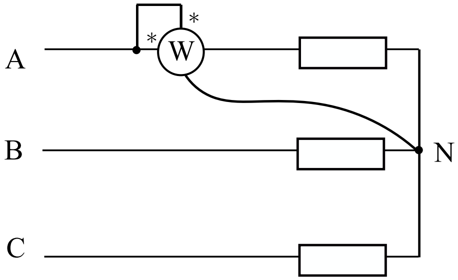
Wattmeters incorporate two measuring circuits: the current coil and the voltage coil.

A sign * marks the terminals of the current and voltage coils of the same polarity



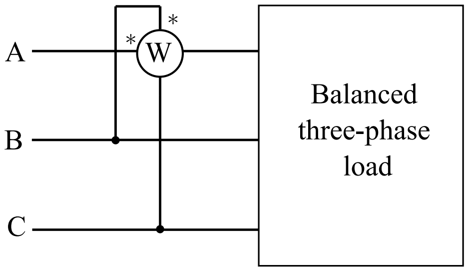
$$W = I_A \cdot U_{AB} \cdot \cos(\widehat{\underline{\mathbf{U}}_{AB}\underline{\mathbf{I}}_A})$$

Measure of the active power in systems with accessible neutral point



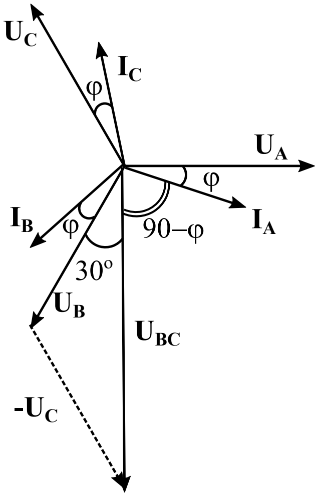
$$W = I_A \cdot U_{AN} \cdot \cos(\widehat{\underline{\mathbf{U}}_{AN}\underline{\mathbf{I}}_A}) = U_{Ph} \cdot I_{Ph} \cdot \cos\varphi = \frac{P}{3}$$

Measure of the reactive power of a three-phase system with one wattmeter

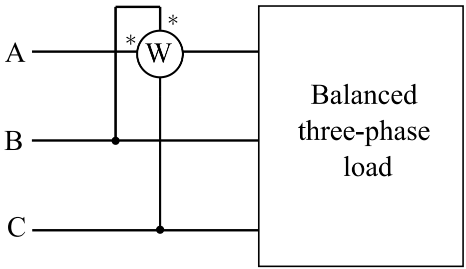


$$W = U_{BC} \cdot I_A \cdot \cos(\widehat{\underline{\mathbf{U}}_{BC}\underline{\mathbf{I}}_A})$$

Angle $\widehat{\underline{\mathbf{U}}_{BC}\underline{\mathbf{I}}_{A}}$

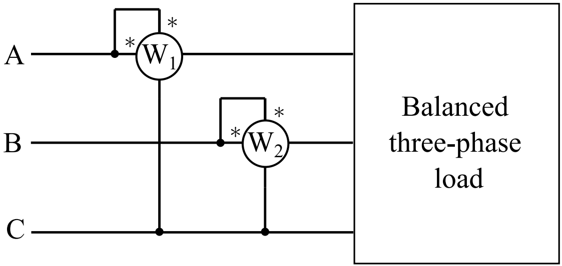


Measure of the reactive power of a three-phase system with one wattmeter



$$W = U_L \cdot I_L \cdot \cos(\widehat{\underline{\mathbf{U}}_{BC}\underline{\mathbf{I}}_A}) = U_L \cdot I_A \cdot cos(90 - \varphi) = U_L \cdot I_A \cdot sin(\varphi) = \frac{Q}{\sqrt{3}}$$

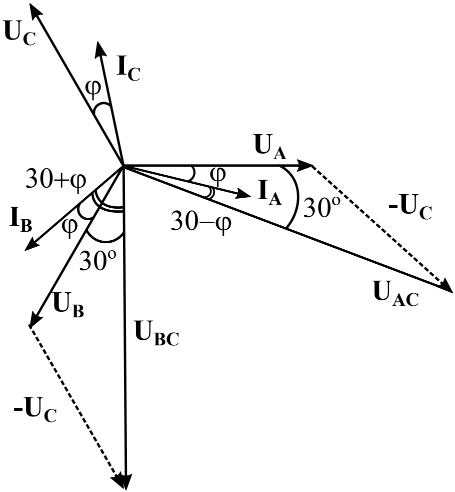
The two wattmeters method



$$W_1 = U_{AC} \cdot I_A \cdot \cos(\widehat{\underline{\mathbf{U}}_{AC}\underline{\mathbf{I}}_A})$$

$$W_2 = U_{BC} \cdot I_B \cdot \cos(\widehat{\underline{\mathbf{U}}_{BC}\underline{\mathbf{I}}_B})$$

Angles $\widehat{\underline{\mathbf{U}}_{AC}\underline{\mathbf{I}}_{A}}$ and $\widehat{\underline{\mathbf{U}}_{BC}\underline{\mathbf{I}}_{B}}$



The two wattmeters method

$$W_1 = U_L \cdot I_L \cdot \cos(\widehat{\underline{\mathbf{U}}_{AC}\underline{\mathbf{I}}_A}) = U_L \cdot I_L \cdot \cos(30 - \varphi) = U_L \cdot I_L \cdot (\frac{\sqrt{3}}{2}\cos\varphi + \frac{1}{2} \cdot \sin\varphi)$$

$$W_2 = U_L \cdot I_L \cdot \cos(\widehat{\underline{\mathbf{U}}_{BC}\underline{\mathbf{I}}_B}) = U_L \cdot I_L \cdot \cos(30 + \varphi) = U_L \cdot I_L \cdot (\frac{\sqrt{3}}{2}\cos\varphi - \frac{1}{2} \cdot \sin\varphi)$$

The active and reactive power of the three phase system can be obtained as the sum and the difference of the measures of the two wattmeters.

$$W_1 + W_2 = \sqrt{3} \cdot U_L \cdot I_L \cdot \cos\varphi = P$$

$$W_1 - W_2 = \cdot U_L \cdot I_L \cdot \sin\varphi = \frac{Q}{\sqrt{3}}$$