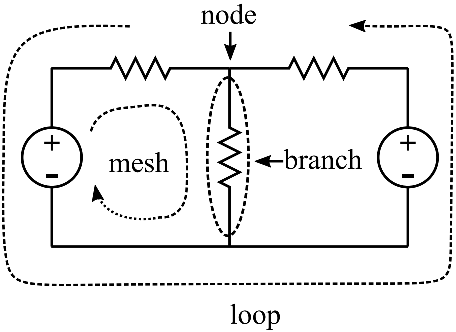
Module 2: Analysis of DC circuits

Electrical power engineering fundamentals

Belén García
Electrical Engineering Department
Universidad Carlos III de Madrid

Definitions

- **Branch**: Part of the circuit with two terminals
- **Node**: Junction of two or more branches
- **Loop**: Close path in a circuit
- **Mesh**: Close path in a circuit that does not have any other close path inside it



Systematic methods of circuit analysis

The application of systematic methods allows us to find the voltages and currents of complex circuits in a simple manner.

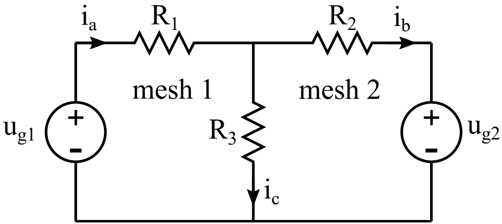
We will study four different methods and will apply them to DC circuits:

1. Mesh current analysis
2. Nodal voltage analysis
3. Superposition principle
4. Thevenin's equivalent

Mesh current method

Mesh current method is based on applying KVL to every mesh of a circuit

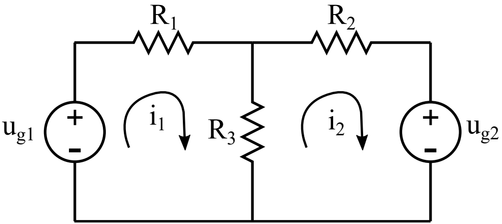
Imagine that we want to solve the circuit of the figure:



Our goal is to calculate the **branch currents** of the circuit: ia, ib, ic

Mesh currents method

1. Define a "Mesh current" for each mesh of the circuit



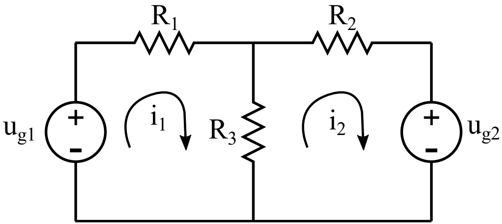
2. Apply KVL to each mesh of the circuit considering that the current that flows through each mesh is the correspondent mesh current

$$\sum_{k} u_k = 0$$

Mesh currents method

To write the equations we consider:

- Voltage drops $+$. Voltage rises -
- Voltage always drops across resistors



Mesh 1: $-u_{g1} + R_1 \cdot i_1 + R_3 \cdot (i_1 - i_2) = 0$

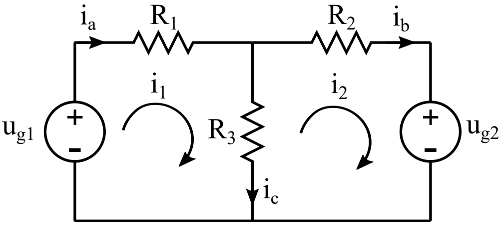
Mesh 2: $R_2 \cdot i_2 + R_3 \cdot (i_2 - i_1) + u_{g2} = 0$

Mesh currents method

3. Solve the equations to find the mesh currents. It is advisable to write the equations in matrix form:

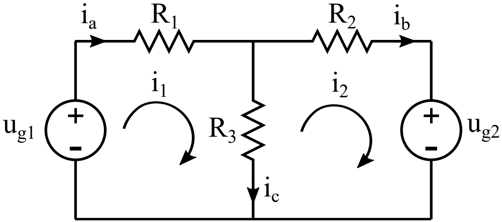
$$\begin{pmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} u_{g1} \\ -u_{g2} \end{pmatrix}$$

4. Calculate the branch currents using the values of the mesh currents.



$$i_a = i_1 \qquad i_b = i_2 \qquad i_c = i_1 - i_2$$

Mesh equations in matrix form

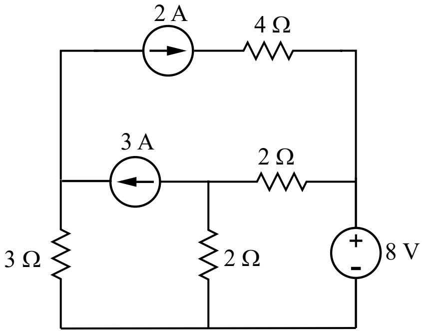


$$[\mathfrak{R}] \cdot [I_{mesh}] = [U_g]$$

R_{ii} = Resistance in mesh i
R_{ij} = Resistance shared by mesh i and j

$$[Ug] = \begin{bmatrix} \sum \text{Voltage rise across sources in mesh 1} \\ \sum \text{Voltage rise across sources in mesh 2} \end{bmatrix}$$

Application of mesh analysis in circuits with current sources



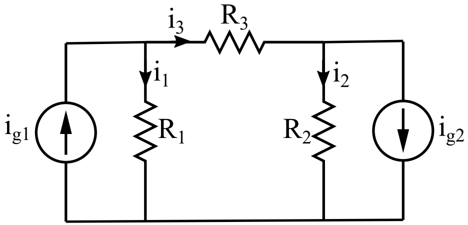
We don't know the voltage drop across a current source beforehand

Situation	Approach
Real current source	Transform into real voltage source
Ideal c.s. part of one mesh	imesh = ig
Ideal c.s. shared by two mesh	Supermesh or ux

Node voltage method

Node voltage method is based in the application of KCL to every independent node of a circuit.

Imagine that we want to solve the circuit of the figure:

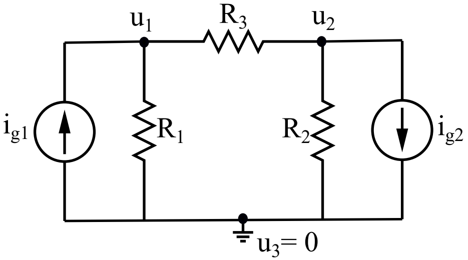


Our goal is to calculate the **branch currents** of the circuit: i1, i2, i3

Node voltage method

To solve a circuit with nodal analysis we must follow the next steps:

1. Identify the independent nodes of the circuit, i.e. the nodes that have a different voltage level and assign them a "nodal voltage"

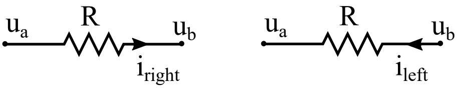


2. Set a "reference node" in which u=0 and then apply KCL to the remaining nodes of the circuit, considering that the voltage level of each node is the correspondent nodal voltage.

Node voltage method

To write the equations we consider:

- Currents flowing out of the node +. Currents flowing into the node - (arbitrary criteria)
- Currents through resistors are calculated as a function of the nodal voltages at their two terminals.

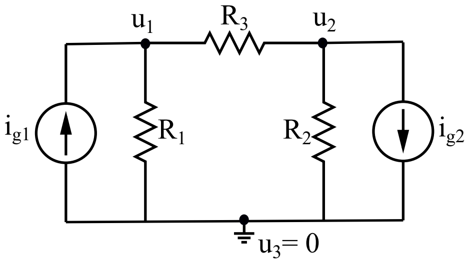


$$i_{right} = \frac{u_a - u_b}{R} \qquad i_{left} = \frac{u_b - u_a}{R}$$

- We consider that the currents that flow through resistors are always flowing out of the node (arbitrary criteria)

Node voltage method

Then, the nodal equations are:



$$\textbf{Equation for node 1:} - i_{g1} + \frac{u_1}{R_1} + \frac{u_1 - u_2}{R_3} = 0$$

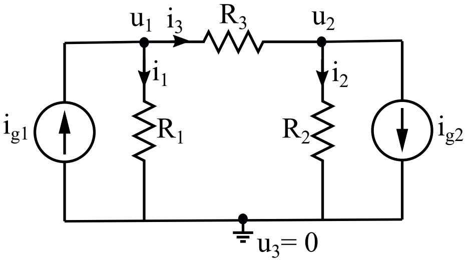
$$\textbf{Equation for node 2:}\ i_{g1} + \frac{u_2}{R_2} + \frac{u_2 - u_1}{R_3} = 0$$

Node voltage method

3. Solve the equations to find the nodal voltages.

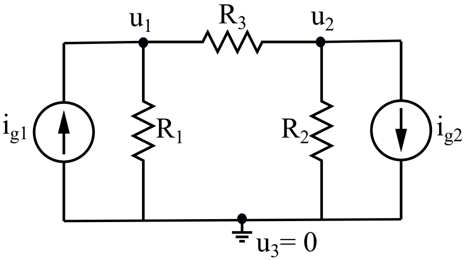
$$\begin{pmatrix} \frac{1}{R_1} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_2} + \frac{1}{R_3} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} i_{g1} \\ -i_{g2} \end{pmatrix}$$

4. Calculate the branch currents.



$$i_1 = \frac{u_1}{R_1} \qquad i_2 = \frac{u_2}{R_2} \qquad i_3 = \frac{u_1 - u_2}{R_3}$$

Nodal equations in matrix form



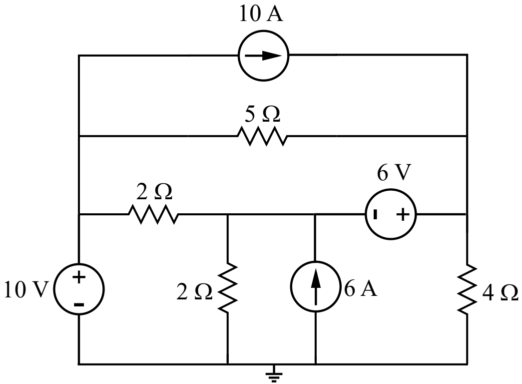
$$[\mathfrak{G}] \cdot [U_{node}] = [I_g]$$

G_{ii} = Conductance connected to node i

G_{ij} = Conductance between nodes i and j

$$[Ig] = \begin{bmatrix} \sum \text{Current injected by current sources into node 1} \\ \sum \text{Current injected by current sources into node 2} \end{bmatrix}$$

Nodal analysis in circuits with voltage sources

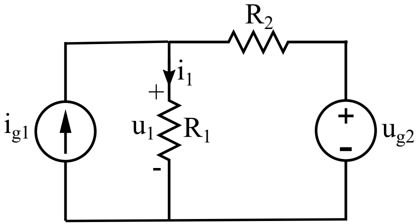


We don't know the current flowing through the voltage drop beforehand

Situation	Approach
Real voltage source	Transform into real current source
Ideal v.s. node-ground	voltage of the node= ug
Ideal v.s. between two nodes	Supernode or ix

Superposition principle

The response of a linear circuit subjected to several excitation sources acting simultaneously equals the sum of the responses of the circuits when the sources act separately.

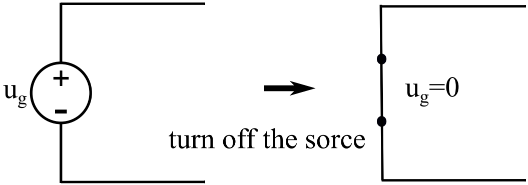


The current i_1 is the sum of the current that flows through the resistor if only the current source is working and the current that flows though it if only the voltage source is working.

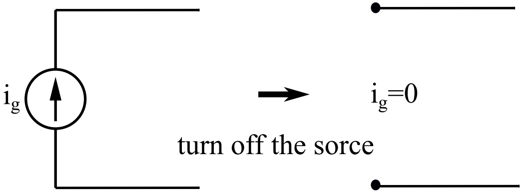
The same happens for voltage u_1

Elimination of sources

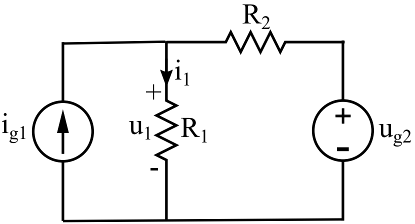
Voltage source

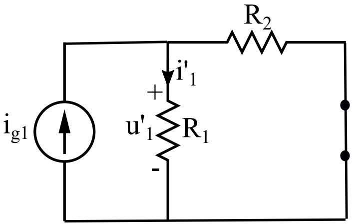


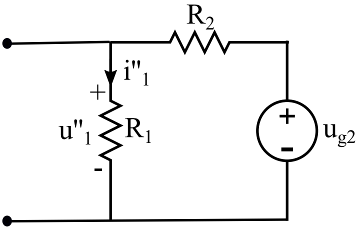
Current source



Superposition principle





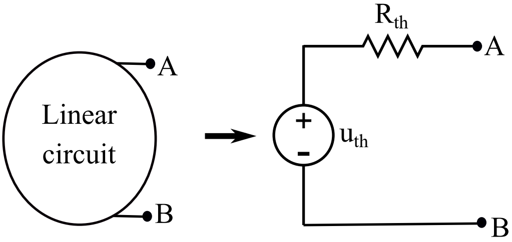


$$i_1 = i_1' + i_1''$$

$$u_1 = u_1' + u_1''$$

Thevenin's theorem

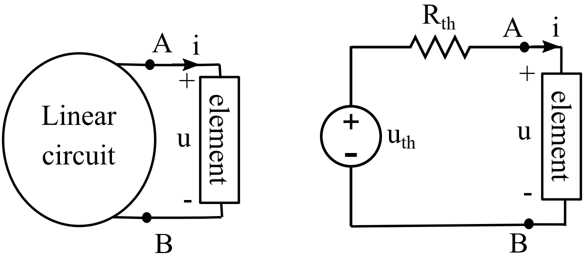
Any linear circuit seen from two terminals can be replaced by a simplified circuit consisting of an ideal voltage source in series with a resistor.



Thevenin's equivalent

What is the meaning of equivalent?

Any element connected between A and B would have identical behaviour if the initial circuit is considered or if the equivalent circuit is considered.

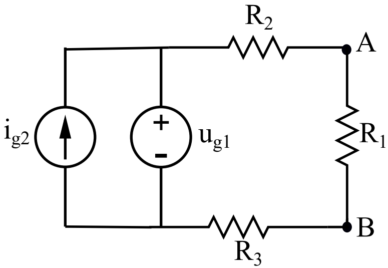


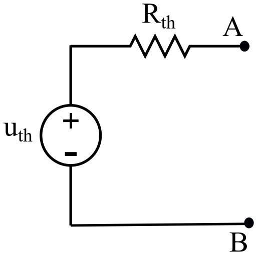
The current flow and the voltage drop across the element is identical in both cases

Determination of the Thevenin equivalent

Calculating the Thevenin equivalent of the circuit means determining what couple of values u_{th} and R_{th} make the behaviour of the simplified circuit represented below identical to the one of the initial circuit, from the point of view of the couple of terminals A B.

Example



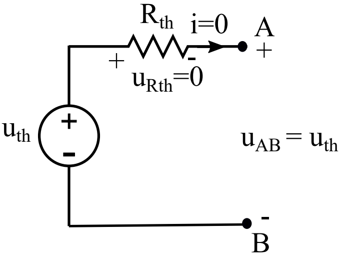


Determination of the Thevenin equivalent

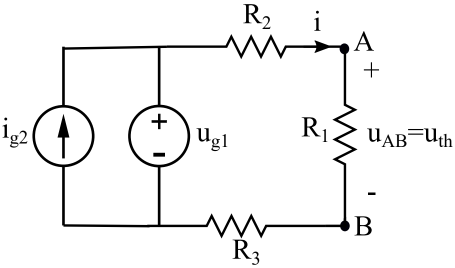
To calculate u_{th} and R_{th} we evaluate how both circuits behave under two particular situations:

1. The behaviour of the two circuits when we leave A B in open circuit (this is the same than connecting a resistance of value infinite between A and B)
2. The behaviour of the two circuits when we place a short circuit between A and B (this is the same than connecting a resistor of value zero)

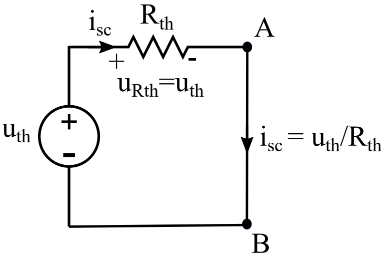
Calculation of u_{th}: open circuit analysis



The analysis of the original circuit without connecting anything between A and B provides the value of u_{th}

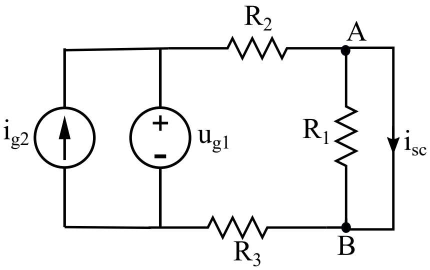


Calculation of R_{th}: short circuit analysis



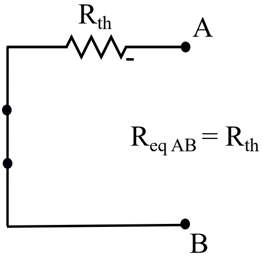
$$R_{th} = \frac{u_{th}}{i_{sc}}$$

As both circuits are equivalent i_{sc} is the current flowing through a short-circuit placed between A and B



Alternative method to calculate R_{th}: passive circuit

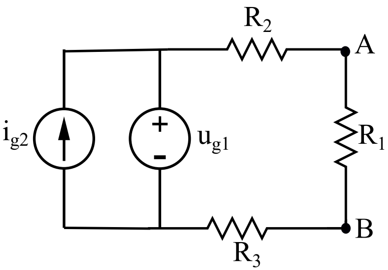
Passivize the circuit and calculate the equivalent resistance between terminals A and B.

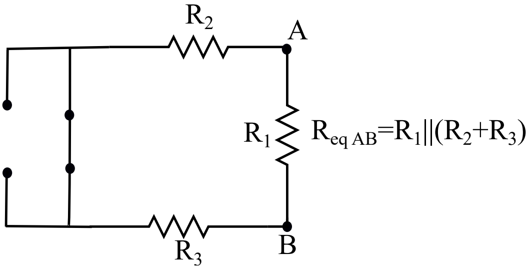


The equivalent resistance between terminals A and B is R_{th}

Alternative method to calculate R_{th}: passive circuit

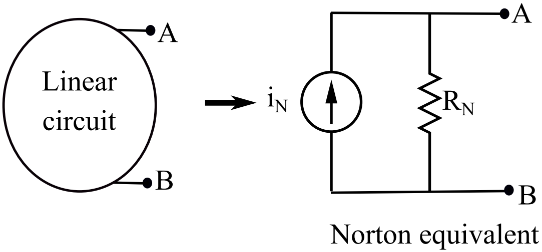
Passivize the circuit: voltage sources turned into short circuits, current sources into open circuits.





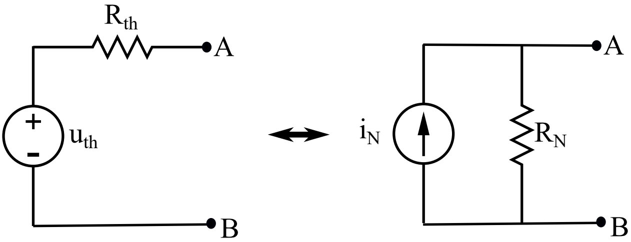
Norton's theorem

Any linear circuit can be replaced by an ideal current source in parallel with a resistor.



Calculation of Norton equivalent

Sources transformation rules can be applied to derive the Norton equivalent from the Thevenin equivalent.

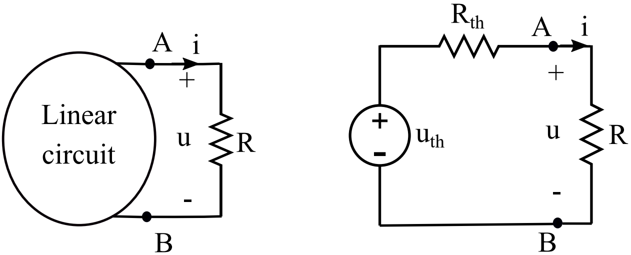


$$R_N = R_{th} \qquad i_N = \frac{u_{th}}{R_{th}}$$

Note that $i_N = i_{sc}$

Maximum transference of power

In certain applications we want to determine the value of the resistor that extract the maximum amount of power from a circuit (i.e. design of antennas)



If we redraw the circuit as its Thevenin equivalent we can calculate the R that absorbs the maximum power from the circuit as:

$$p = R \cdot i^2 = R \cdot \frac{u_{th}^2}{(R_{th} + R)^2} \qquad \frac{dp}{dR} = 0 \qquad R = Rth$$