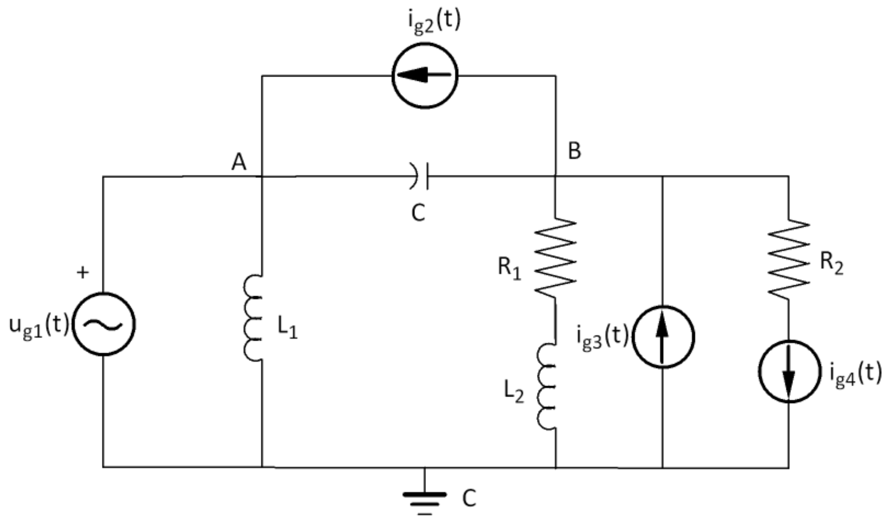
Electrical Power Engineering Fundamentals
Second partial exam, April 12th 2018

1. Solve the following circuit in the frequency domain using **nodal analysis** and obtain the phasors current at each branch.



R1=3 Ω; R2=1 Ω; L1=0.01 H; L2=0.02 H; C=0.01 F

$u_{g1}(t) = \sqrt{2} \cdot 100 \cos(100t) V$

$i_{g2}(t) = \sqrt{2} \cdot 10 \cos(100t + 90) A$

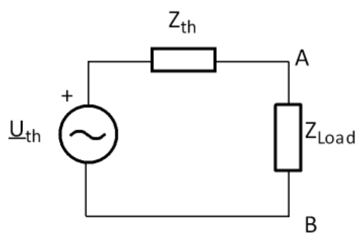
$i_{g3}(t) = \sqrt{2} \cdot 5 \cos(100t) A$

$i_{g4}(t) = \sqrt{2} \cdot 3 \cos(100t + 90) V$

2. Calculate the power balance of the circuit

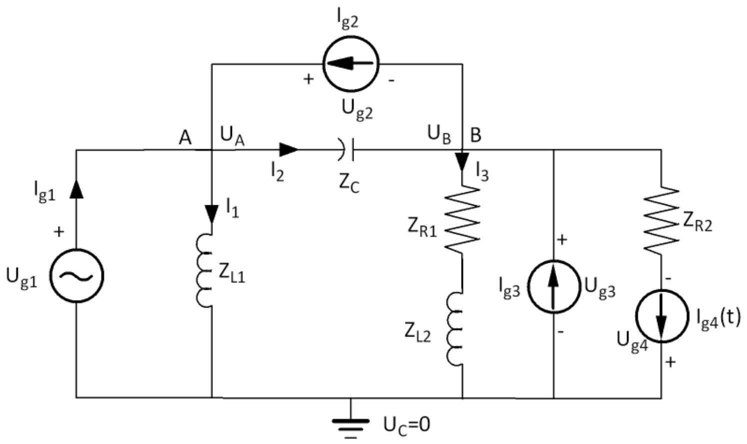
3. Calculate the Thevenin equivalent of the circuit at terminals AB.

4. We connect an impedance $Z_{load} = R + jX\Omega$ at terminals AB as shown in the figure. Calculate the value of Zload knowing that the complex power generated by the Thevenin source in this situation is S=50+150j.



SOLUTION

a) We move to the frequency domain ($\omega = 100 rad/s$), obtaining the impedances of the pasive elements and the phasors that represent the different currents and voltages.



Impedances:

$$Z_{R1} = 3\Omega \qquad Z_{R2} = 1\Omega \qquad Z_{L1} = j\Omega \qquad Z_{L2} = 2j\Omega \qquad Z_C = -j\Omega$$

Phasors:

$$\underline{U_{g1}} = 100V \qquad \underline{I_{g2}} = 10jA \qquad \underline{I_{g3}} = 5A \qquad \underline{I_{g4}} = 3jA$$

Nodal equations for nodes A and B:
Node A:

$$\underline{U_A} = \underline{U_{g1}}$$

$$\frac{\underline{U_B} - \underline{U_A}}{Z_C} + \frac{\underline{U_B}}{Z_R + Z_{L2}} + \underline{I_{g2}} - \underline{I_{g3}} + \underline{I_{g4}} = 0$$

Solving the equations we find the nodal voltages:

$$\underline{U_A} = 100V$$

$$\underline{U_B} = 97.2 + 20.6jV = 99.36_{11.96^o}A$$

Now we calculate the currents that flow through the different branches:

$$\underline{I_1} = \frac{\underline{U_A}}{Z_{L1}} = -100jA$$

$$\underline{I_2} = \frac{\underline{U_A} - \underline{U_B}}{Z_C} = 20.6 + 2.8jA = 20.79_{7.74^o}A$$

$$\underline{I_3} = \frac{\underline{U_B}}{Z_{R1} + Z_{L2}} = 25.6 - 10.2jA = 27.56_{-21.72^o}A$$

$$\underline{I_{g1}} = \underline{I_1} + \underline{I_2} - \underline{I_{g2}} = 20.6 - 107.2j = 109.16_{-79.12^o}A$$

b) Power balance

Active and reactive power of pasive elements

$$P_{loads} = Z_{R1} \cdot I_3^2 + Z_{R2} \cdot I_{g4}^2 = 2287.2W$$

$$Q_{loads} = X_{L1} \cdot I_1^2 + X_{L2} \cdot I_3^2 - |X_C| \cdot I_2^2 = 11087VAr$$

$$S_{loads} = 2287.2 + j11087VA$$

Complex power of sources:

$$S_{Ug1} = \underline{U_{g1}} \cdot \underline{I_{g1}}^* = 2060 + 10720jVA$$

$$S_{Ig2} = (\underline{U_A} - \underline{U_B}) \cdot \underline{I_{g2}}^* = -206 - 28jVA$$

$$S_{Ig3} = \underline{U_{g1}} \cdot \underline{I_{g1}}^* = 486 + 103jVA$$

$$S_{Ig4} = (-\underline{U_B} - (Z_{R2} \cdot \underline{I_{g4}}) \cdot \underline{I_{g4}}^* = -52.8 + 291.6jVA$$

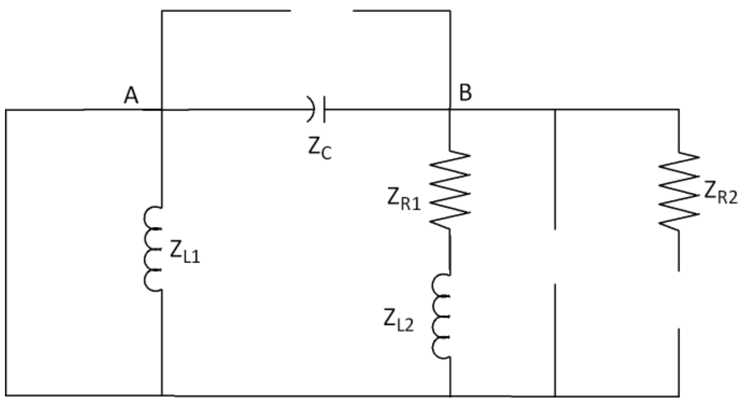
$$S_{generators} = S_{Ug1} + S_{Ig2} + S_{Ig3} + S_{Ig4} = 2287.2 + 11087jVA$$

$$S_{generators} = S_{loads}$$

c) Thevenin equivalent

$$\underline{Uth} = \underline{U_{AB}} = \underline{U_A} - \underline{U_B} = 2.8 - 20.6jV = 20.7894_{-82.2597^o}$$

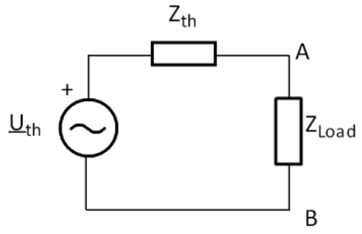
We can obtain Thevenin impedance pasivizing the circuit and finding the equivalent impedance of the resulting net from terminals AB:



The equivalent impedance is:

$$Zth = Z_C||(Z_{R1} + Z_{L2}) = \frac{Z_C \cdot (Z_{R1} + Z_{L2})}{Z_C + Z_{R1} + Z_{L2}} = 0.3 - 1.1j\Omega = 1.14_{-74.74^o}\Omega$$

d) The resulting equivalent is:



The current supplied by the Thevenin source when the impedance is connected is

$$\underline{I}^* = \frac{S_{gth}}{\underline{U_{th}}} = \frac{50 + 150j}{2.8 - 20.6j} = -6.82 - 3.35jA$$

$$\underline{I} = -6.82 + 3.35jA$$

Voltage drop at Zload

$$\underline{U_{Zload}} = \underline{U_{th}} - \underline{I} \cdot Z_{th} = 8.54 - 27.10jV$$

Then:

$$Z_{load} = \frac{\underline{U_{Zload}}}{\underline{I}} = 0.56 + 3.69j\Omega$$