Electrical Power Engineering Fundamentals

AC CIRCUITS (15th November 2019)

In the circuit below:

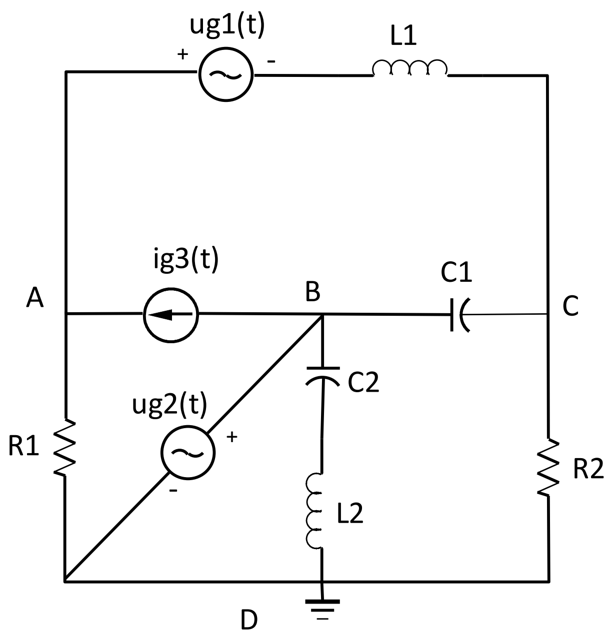
$u_1(t) = \sqrt{2} \cdot 1{,}25 \cdot cos(100t)\ V$

$u_2(t) = \sqrt{2} \cdot 10 \cdot cos(100t)\ V$

$i_3(t) = \sqrt{2} \cdot 8 \cdot\ cos(100t - 90)$ A

$R1 = 0.25\ \Omega;\ \ R2 = \ 1\ \Omega, L1 = 2.5\ mH;\ \ L2 = 5\ mH;\ \ C1 = 0.05\ F;\ C2 = \ 0.01\ F$

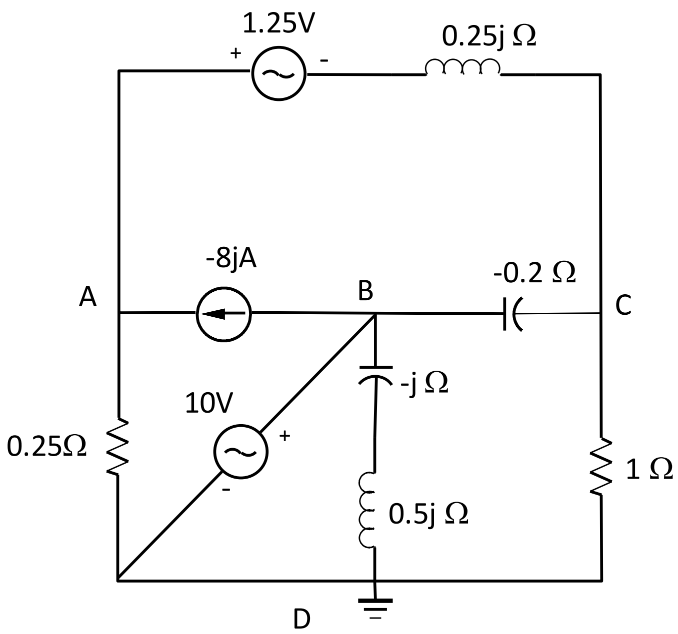
a) Write the nodal equations for nodes A, B and C taking D as reference node. Express the equations as a matrix system. Solve the system and calculate the nodal voltages in phasor form
b) Calculate the complex power supplied by the voltage source ug1
c) Calculate the Thevenin's equivalent of the circuit at terminals C D including all the elements of the circuit in the equivalent. Draw the equivalent
d) Calculate the complex power absorbed by an impedance Z_{load}=3+7j connected between terminals C D.



Solution

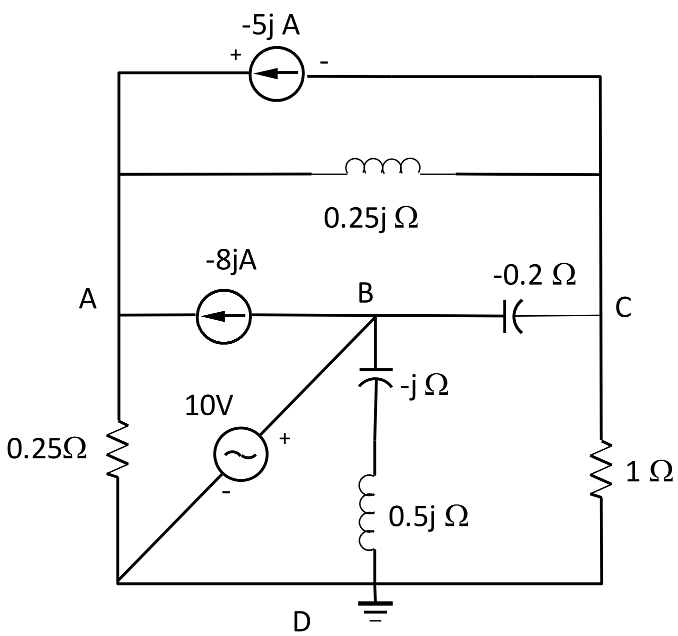
We transform the circuit into the frequency domain using the equations for the impedances:

ZR=R; ZL=jωL; ZC=-j/ωC



We can transform the real voltage source into a real current source to facilitate the analysis of the system with nodal method

Ig1= Ug1/Zl1=-5j A



Nodal equations in matrix form

$$\begin{pmatrix} \frac{1}{R1}+\frac{1}{ZL1} & 0 & -\frac{1}{ZL1} \\ 0 & 1 & 0 \\ -\frac{1}{ZL1} & -\frac{1}{ZC1} & \frac{1}{ZL1}+\frac{1}{ZC1}+\frac{1}{ZR2} \end{pmatrix} \begin{pmatrix} U1 \\ U2 \\ U3 \end{pmatrix} = \begin{pmatrix} Ig1+Ig3 \\ Ug2 \\ -Ig1 \end{pmatrix}$$

$$\begin{pmatrix} 4-4j & 0 & 4j \\ 0 & 1 & 0 \\ 4j & -5j & 1+1j \end{pmatrix} \begin{pmatrix} U1 \\ U2 \\ U3 \end{pmatrix} = \begin{pmatrix} -13j \\ 10 \\ 5j \end{pmatrix}$$

Solving

$\underline{U}$A=9.7083 - 0.5417i=9.7234 $\underline{/-3.1934\ º}$
$\underline{U}$B=10V
$\underline{U}$C= 7.0000 + 9.1667i= 11.5338 $\underline{/52.6333\ º}$

b)
$\underline{U}$l1=($\underline{U}$C-$\underline{U}$A)+$\underline{U}$g1

$\underline{U}$l1= -1.4583 + 9.7083i=9.8173 |98.54º A

$\underline{I}$ca=$\underline{U}$l1/Zl1

$\underline{I}$ca = 38.8333 + 5.8333i=39.269 | 8.5428A

Sg1=Sg1*$\underline{I}$ca*

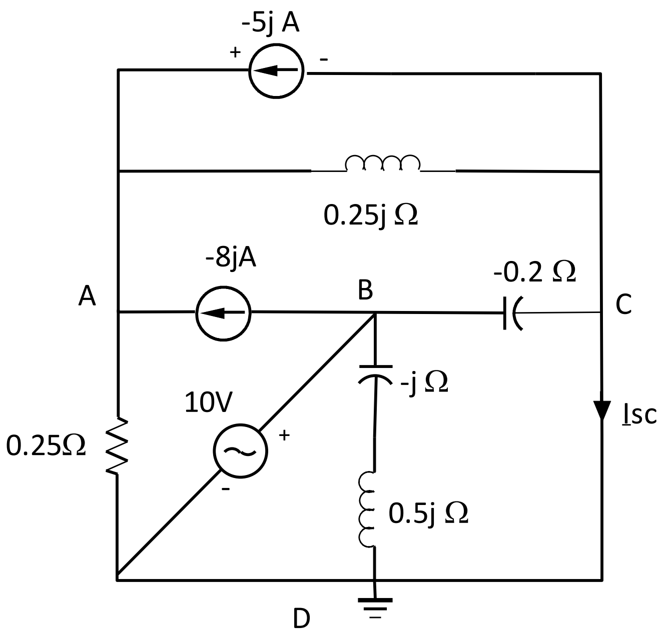
Sg1 =48.5417 - 7.2917i

c) Thevenin's voltage

$\underline{U}$th=$\underline{U}$C=11.5338 $\underline{/52.6333\ º}$ V

uth=uD= 7.0000 + 9.1667i= 11.5338 | 52.6333º

% Impedance with short circuit current



-5j A
+
-
0.25j Ω
-8jA
A
B
-0.2 Ω
C
-j Ω
10V
+
-
0.25Ω
Isc
0.5j Ω
D