

Electrical power engineering fundamentals

Partial exam. 25th November 2020

Instructions Solve the problems using the methods indicated in the problem statements and write a summary of your results in this paper. Only the solutions obtained with these methods will be graded.

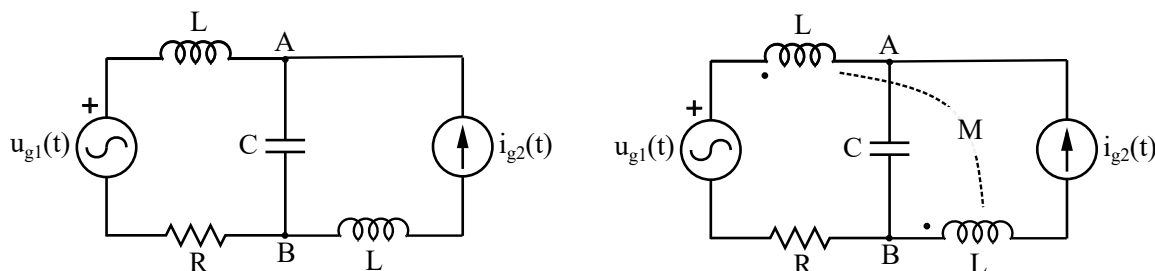
Name.....

AC circuits

Given that the values of the parameters are $R = 4$, $L = 0.1H$, $C = 25mF$

$$u_{g1}(t) = \sqrt{2} \cdot 13 \cdot \cos 10tV$$

$$i_{g2}(t) = \sqrt{2} \cdot 2 \cdot \cos(10t + 90)A$$



a) Find the Thevenin equivalent in the circuit of the figure on the left between A and B **including all the elements of the circuit in it**. Draw the equivalent indicating the obtained values for the parameters. (6 points)

b) Recalculate the Thevenin equivalent if the two inductors of the circuit are coupled as in the figure on the right given that $M = 0.5H$. (4 points)

Solution

a) Circuit in the frequency domain $\omega = 10rad/s$

$$Z_R = 4\Omega \quad Z_L = j\Omega \quad Z_C = -4j\Omega$$

$$\underline{U}_{g1} = 13V \quad \underline{I}_{g2} = 2jA$$

Firstly we calculate Thevenin voltage with the open circuit analysis: $\underline{U}_{th} = \underline{U}_{AB}$

Applying mesh currents method:

$$-\underline{U}_{g1} + Z_L \cdot \underline{I}_1 + Z_C \cdot (\underline{I}_1 - \underline{I}_2) + Z_R \cdot \underline{I}_1 = 0$$

$$\underline{\mathbf{I}}_2 = -\underline{\mathbf{I}}_{g2}$$

$$\underline{\mathbf{I}}_1 = 0.8 + 0.6j = 1\angle 36.87A \quad \underline{\mathbf{I}}_2 = -2jA$$

$$\underline{\mathbf{U}}_{th} = \underline{\mathbf{U}}_{AB} = Z_C \cdot (\underline{\mathbf{I}}_1 - \underline{\mathbf{I}}_2) = 10.4 - 3.2j = 10.88\angle -17.10V$$

To calculate Thevenin impedance we passivize the circuit and calculate Z_{eqAB}

$$Z_{th} = Z_{eqAB} = (Z_R + Z_L) || Z_C = 2.56 - 2.08j\Omega$$

b) We repeat the same process considering that the two inductors are coupled. $Z_M = j \cdot \omega \cdot M = 5j\Omega$

Mesh equations:

$$-\underline{\mathbf{U}}_{g1} + Z_L \cdot \underline{\mathbf{I}}_1 - Z_M \cdot \underline{\mathbf{I}}_2 + Z_C \cdot (\underline{\mathbf{I}}_1 - \underline{\mathbf{I}}_2) + Z_R \cdot \underline{\mathbf{I}}_1 = 0$$

$$\underline{\mathbf{I}}_2 = -\underline{\mathbf{I}}_{g2}$$

$$\underline{\mathbf{I}}_1 = 2.4 + 1.8j = 3\angle 36.87A \quad \underline{\mathbf{I}}_2 = -2jA$$

$$\underline{\mathbf{U}}_{th} = \underline{\mathbf{U}}_{AB} = Z_C \cdot (\underline{\mathbf{I}}_1 - \underline{\mathbf{I}}_2) = 15.2 - 9.6j = 17.98\angle -32.27V$$

To calculate Thevenin's impedance we should consider a short-circuit between A and B and calculate $\underline{\mathbf{I}}_{sc}$

Mesh equations when a short-circuit is placed between A and B:

$$-\underline{\mathbf{U}}_{g1} + Z_L \cdot \underline{\mathbf{I}}_1 - Z_M \cdot \underline{\mathbf{I}}_2 + Z_R \cdot \underline{\mathbf{I}}_1 = 0$$

$$\underline{\mathbf{I}}_2 = -\underline{\mathbf{I}}_{g2}$$

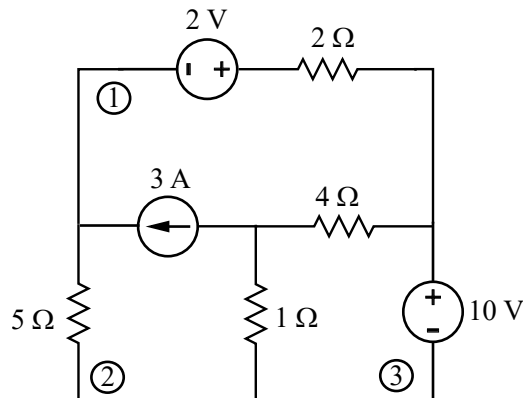
$$\underline{\mathbf{I}}_1 = 5.41 + 1.35j = 5.58\angle -14.04A \quad \underline{\mathbf{I}}_2 = -2jA$$

$$\underline{\mathbf{I}}_{sc} = \underline{\mathbf{I}}_1 - \underline{\mathbf{I}}_2 = 5.41 + 0.65j = 5.45\angle 6.82A$$

$$Z_{th} = \frac{\underline{\mathbf{U}}_{th}}{\underline{\mathbf{I}}_{sc}} = 2.56 - 2.08j\Omega$$

DC circuits

In the DC circuit of the figure:



- Write the equations for the **mesh current method** for the circuit in the figure. (Label the mesh currents with the names provided in the figure and take the mesh currents in clockwise direction). (2.5 points)
- Write the mesh equations in matrix form. (2 points)
- Solve the equations and provide the values in the circuit figure indicating their value and direction (2.5 points)
- Do a power balance of the circuit and write the total power delivered by sources and the total power absorbed by resistors (3 points)

Solution

We write the mesh equations using the supernode approach:

$$\begin{aligned} -2 + 2 \cdot i_1 + 10 + 5 \cdot i_2 &= 0 \\ 10 + 1 \cdot (i_3 - i_2) + 4 \cdot (i_3 - i_1) &= 0 \\ i_1 - i_2 &= 3 \end{aligned}$$

$$\begin{pmatrix} 2 & 5 & 0 \\ -4 & -1 & 5 \\ 1 & -1 & 5 \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -8 \\ -10 \\ 3 \end{pmatrix}$$

$$i_1 = 1A \quad i_2 = -2A \quad i_3 = -1.6A$$

We calculate the branch currents and the power absorbed by resistors:

$$P_R = \sum R \cdot i^2 = 49.2W$$

Power delivered by sources:

$$P_g = \sum u \cdot i = 2 \cdot 1 + 10 \cdot 1.6 + 3 \cdot 10.4 = 49.2W$$