Electrical power engineering fundamentals

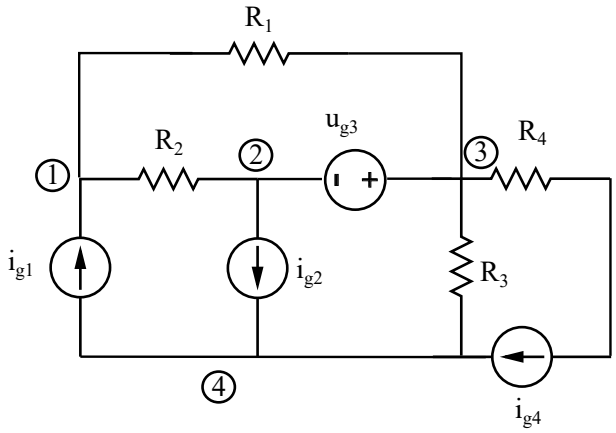
Partial exam. 25th November 2020

Instructions Solve the problems using the methods indicated in the problem statements and write a summary of your results in this paper. Only the solutions obtained with these methods will be graded.

Name.....

DC Circuits

Given that the values of the parameters are $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $R_3 = 3\Omega$, $R_4 = 4\Omega$, $i_{g1} = 1A$, $i_{g2} = 2A$, $u_{g3} = 3V$, $i_{g4} = 4A$



a) Apply nodal analysis to solve the circuit taking 4 as reference node. Write the nodal equations and solve them to find the node voltages (5 points)

b) Calculate the power delivered by ig4 (2 points)

c) Calculate the Thevenin equivalent between nodes 1 and 2 including all the elements of the circuit in the equivalent. Draw the equivalent indicating the values of the parameters (3 points)

Solution

a) Nodal equations:

$$-i_{g1} + \frac{u_1 - u_3}{R_1} + \frac{u_1 - u_2}{R_2} = 0$$

$$\frac{u_2 - u_1}{R_2} + i_{g2} + i_x = 0$$

$$\frac{u_3 - u_1}{R_1} - i_x + i_{g4} + \frac{u_3}{R_3} = 0$$

$$u_3 - u_2 = u_{g3}$$

$$u_1 = -15.33V \qquad u_2 = -18V \qquad u_3 = -15V \qquad i_x = -0.67A$$

b) Power delivered by i_{g4}

$$u_{g4} = -u_3 + u_{R4} = 15 + 16 = 31V$$

$$p_{g4} = u_{g4} \cdot i_{g4} = 31 \cdot 4 = 124W$$

c) Thevenin equivalent:

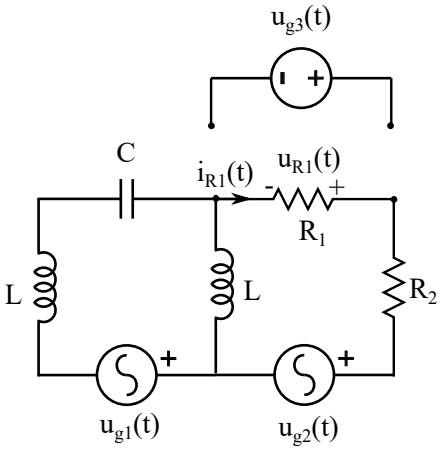
$$u_{th} = u_1 - u_2 = -15.33 - (-18) = 2.67V$$

$$R_{th} = R_1 || R_2 = 0.67\Omega$$

AC circuits

Given that the values of the parameters are $R_1 = 2$, $R_2 = 3$, $L = 0.2H$, $C = 50mF$

$$u_{g1}(t) = \sqrt{2} \cdot 80 \cdot \cos 10tV \qquad u_{g2}(t) = \sqrt{2} \cdot 50 \cdot \cos(10t + 90)V$$



a) Solve the circuit using mesh analysis and calculate the branch currents of the circuit. Write the equations in matrix form and the phasors mesh currents below. (4 points)

b) Do a power balance of the circuit. Write a summary of your results (3 points)

c) If the DC source ug3 is connected between the terminals of R1. Calculate $u_{R1}(t)$ and $i_{R1}(t)$ with the polarities specified in the circuit. $u_{g3}(t) = 18V$. (3 points)

Solution

a) Circuit in the frequency domain $\omega = 10 rad/s$

$$Z_{R1} = 2\Omega \qquad Z_{R2} = 3\Omega \qquad Z_L = 2j\Omega \qquad Z_C = -2j\Omega \qquad \underline{\mathbf{U}}_{g1} = 80V \qquad \underline{\mathbf{U}}_{g2} = 50jA$$

Mesh equations:

$$\begin{pmatrix} Z_L + Z_C + Z_L & -Z_L \\ -Z_L & Z_L + Z_{R1} + Z_{R2} \end{pmatrix} \cdot \begin{pmatrix} \underline{\mathbf{I}}_1 \\ \underline{\mathbf{I}}_2 \end{pmatrix} = \begin{pmatrix} -\underline{\mathbf{U}}_{g1} \\ -\underline{\mathbf{U}}_{g2} \end{pmatrix}$$

$$\underline{\mathbf{I}}_1 = -16 + 30j = 34\angle 118.07A \qquad \underline{\mathbf{I}}_2 = -16 - 10j = 18.87\angle -148A$$

b) Power of loads

$$P_{R1} = Z_{R1} \cdot I_2^2 = 712W$$
$$P_{R2} = Z_{R1} \cdot I_2^2 = 1068W$$
$$Q_L = X_L \cdot I_1^2 + X_L \cdot |\underline{\mathbf{I}}_1 - \underline{\mathbf{I}}_2|^2 = 2312 + 3200 = 5512var$$
$$Q_C = X_C \cdot I_1^2 = -2312var$$
$$S_{loads} = P_{R1} + P_{R2} + j \cdot (Q_L + Q_C) = 1780 + 3200jVA$$

Sources:

$$S_{g1} = \underline{\mathbf{U}}_{g1} \cdot (-\underline{\mathbf{I}}_{g1})^* = 1280 + 2400jVA$$
$$S_{g2} = \underline{\mathbf{U}}_{g2} \cdot (-\underline{\mathbf{I}}_{g2})^* = 500 + 800jVA$$
$$S_{sources} = S_{g1} + S_{g2} = 1780 + 3200jVA$$

c) As u_{g3} is a DC source we should use superposition principle to analyse the response of the circuit to the sources of different frequencies.

First we turn off the source u_{g3} that becomes a short circuit. Then:

$$u'_{R1} = 0 \qquad i'_{R1} = 0$$

Then we turn off sources u_{g1} and u_{g2} and calculate the response of the circuit to u_{g3}

$$u''_{R1} = u_{g3} = 18V \qquad i''_{R1} = -u_{R1}/R1 = -9A$$

Response of the three sources acting simmulateously:

$$u_{R1}(t) = u'_{R1} + u''_{R1} = 18V \qquad i_{R1}(t) = i'_{R1} + i''_{R1} = -9A$$