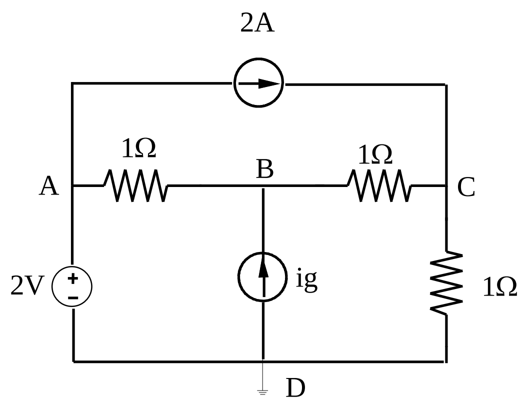
ELECTRICAL POWER ENGINEERING FUNDAMENTALS

FINAL EXAM. ORDINARY CALL (January 10th, 2020)

Exercise 1

For the circuit below:

a) Write the equations for the mesh analysis
b) Knowing that the power delivered by the voltage source is 0, calculate ig
c) Calculate the Thevenin's equivalent of the circuit between B and D including all the elements of the circuit in it except the current source ig.



Solution

a) Mesh equations:

Mesh1: i1=2A
Mesh 2: -2+ (i2-i1)+ux=0
Mesh 3: -ux+(i3-i1)+i3=0
Additional equation: i3-i2=ig

b) Since the voltage source do not supply power i2=0. Then the mesh equations can be simplified to:

i1=2A
-2 -i1+ux=0
-ux+2i3-i1 =0
i3 =ig

Summing equations 2 and 3:

-2-i1+2i3-i1=0 => 2i3-6=0 => i3=ig= 3A

d) We need to remove the source ig from the circuit to calculate Thevenin's equivalent.

uth=u_{BD} in the resulting circuit.

Applying mesh analysis:

i1=2A

2(i2-i1)+i2-2=0

Solving

3i2-2i1-2=0 => i2=2 A;

The current flowing from A to C is 0; uth= u_{BD} =u_{CD} = 2V

To calculate Rth we passivize the circuit and calculate Rth= Req_{CD}=(1+1)||1=2/3 □

Exercise 2

In the circuit below:

$u_1(t) = \sqrt{2} \cdot 10 \cdot cos(100t)\ V$

$u_2(t) = \sqrt{2} \cdot 5 \cdot cos(100t + 90)\ V$

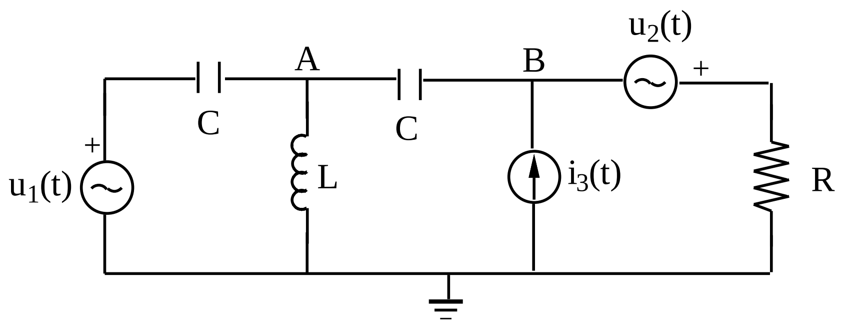
$i_3(t) = \sqrt{2} \cdot 10 \cdot\ cos(100t + 30)$ A

$R = 5\ \Omega;\ \ L = 5\ mH;\ C = 5\ mF$

a) Apply nodal analysis to find the nodal voltages $u_A(t)$ and $u_B(t)$
b) Do a power balance of the circuit
c) We want to add a current source to the circuit, connected between terminals A and B, so that the voltage drop across the resistor R (+ up – down) becomes:

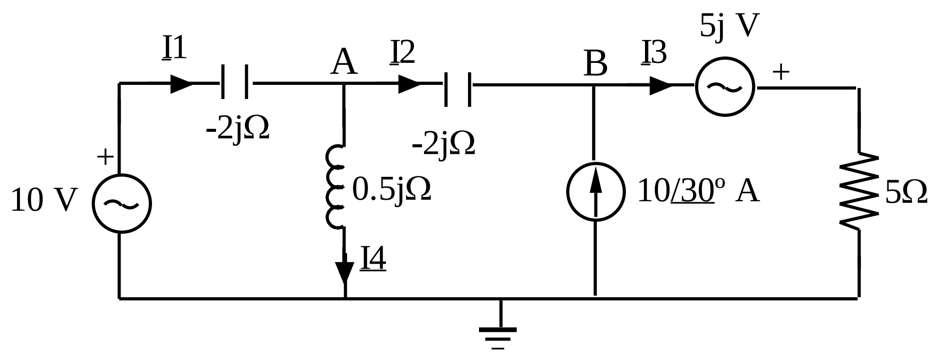
$u_R(t) = 25 + \sqrt{2} \cdot 7 \cdot\ cos(100t - 48)\ A$

Determine the instantaneous value and the polarity of the source.



Solution

a) Circuit in the frequency domain:



Nodal equations:

(To obtain the equations we could redraw the voltage sources as current sources or find the voltage drop across the impedances that are in series with them)

$$\frac{U_A - 10}{-2j} + \frac{U_A}{0.5j} + \frac{U_A - U_B}{-2j} = 0$$

$$\frac{U_B + 5j}{5} - 10/30 + \frac{U_B - U_A}{-2j} = 0$$

$$\begin{pmatrix} \frac{1}{-2j} + \frac{1}{-2j} + \frac{1}{0.5j} & \frac{1}{2j} \\ \frac{1}{2j} & \frac{1}{-2j} + \frac{1}{5} \end{pmatrix} \begin{pmatrix} UA \\ UB \end{pmatrix} = \begin{pmatrix} 5j \\ 10/30 - j \end{pmatrix}$$

Solving:

UA = -7.371+ 5.14 j=8.9869/145.10 V

UB = 4.74 -10.28 j =11.32 /-65.24 V

a) Power balance

Branch currents:

I1 =(10-UA)/(-2j) = 2.57 + 8.68 j = 9.06 / 73.51 A
I2= (UA-UB)/(-2j) = -7.7119 - 6.0565j =9.81 /-141.86
I3= i3= (ub+5j)/5 = 0.95 - 1.06j = 1.41 /-48.09 A
I4=UA/0.5j = 10.28 +14.74 j= 17.97 /55.10 A

Power absorbed by the passive elements

$PR=R·I3^2$=10.08W
$QL=XL*I4^2$ =161.53 var
$QC=XC*I2^2 + XC*I1^2$ = -356.40 var

*S*power generated by the sources:

*S*g1=Ug1·I1*= 25.71 -86.86 j VA
*S*g2= Ug2·I3*= -5.28 + 4.74 j VA
*S*g3=UB·I2*= -10.35 -112.76 j VA

Power balance:

*S*Tg = *S*g1+*S*g2+*S*g3 = 10.08 -194.87 j VA = *S* abs passive

d) Applying the superposition principle we find that in order to obtain

$$u_R(t) = 25 + \sqrt{2} \cdot 7 \cdot\ cos(100t - 48)\ A$$

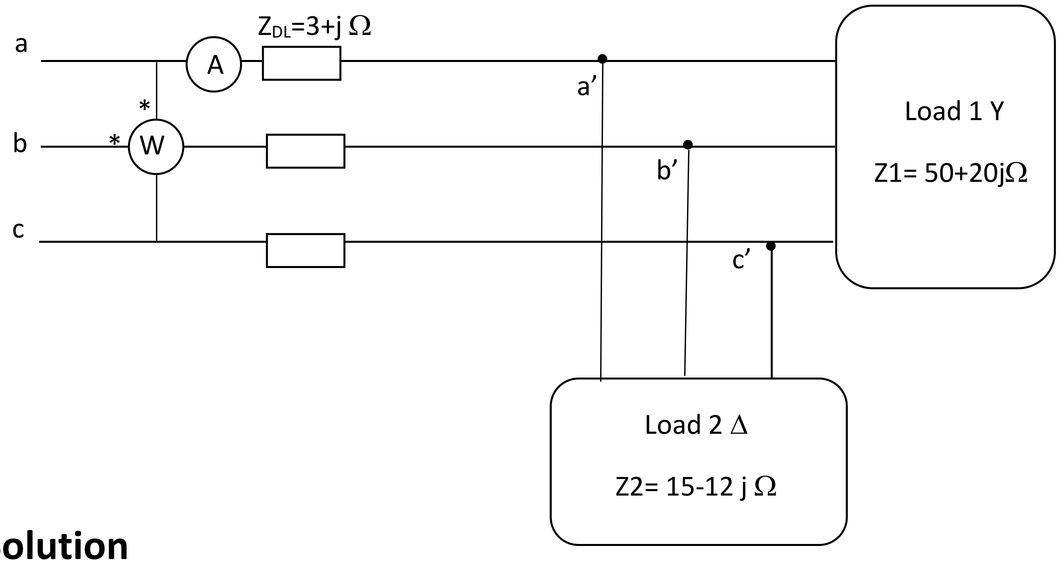
We should connect a DC current source of 5A with polarity arrow to the right.

Exercise 3

The following diagram represents a three-phase system wich supplies energy to two three-phase loads. Load 1 is Y connected and Load 2 is Δ connected. The impedance per phase of each load is indicated in the diagram. The loads are connected to a generator by means of a distribution line with impedance Z_{DL}= 3+j Ω.

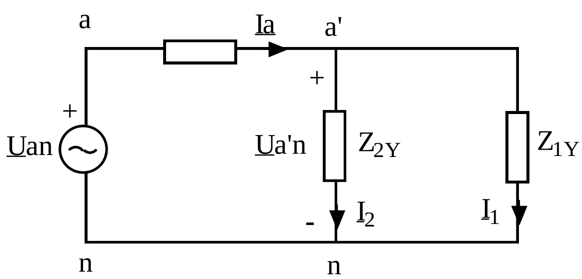
The phasor line voltage at the load end of the system has a constant value Ua'b'=380 /30º V.

a) Draw the one-phase equivalent of the system.
b) Determine the measure of the ammeter
c) Determine the modulus of the line voltage at the generator
d) Calculate the active and reactive power absorbed by load 1 and load 2, the power factor of each load and the power factor of the set formed by the two loads.
e) Calculate the measure of the wattmeter



Solution

a) One phase equivalent



Z_{2Y}=3-4j Ω

Ua'n=380/sqrt(3) = 219.39 /0 V

b) To calculate the line current we can associate the two impedances in parallel
$Zeq=(Z_{1Y}*Z_{2Y})/(Z_{1Y}+Z_{2Y})$= 5.0442 - 3.2856 j= 6.01 $\underline{/-33.05}$ Ω

$\underline{I}a=\underline{U}a'n/Zeq$= 30.54 +19.89 j= 36.44 $\underline{/33.07}$ A

The ammeter measures 36.44 A

Alternatively we could calculate the current flowing through each impedance and sum them to get the total line current:

$\underline{I}1=\underline{U}a'n/Z_{1Y}$= 4.07 $\underline{/-21.80}$ A
$\underline{I}2=\underline{U}a'n/Z_{2Y}$= 34.26 $\underline{/38.66}$ A

$\underline{I}a=\underline{I}1+\underline{I}2$ = 36.44 $\underline{/33.07}$ A

c) $\underline{U}an=\underline{U}a'n+(Z_{DL}\cdot\underline{I}a)$= 291.12 + 90.21 j V = 304.76 $\underline{/17.22}$ V

ULg=sqrt(3) · 304.76 = 527.88 V

d) $P1=3\cdot R1\cdot|I1|^2$ = 2484.73 W
$Q1=3\cdot X1\cdot |I1|^2$ = 993.89 var
pf1= 0.93 ind

(The pf can be calculated with the power triangle, the impedance triangle or the phase shift between the current and voltage)

I2ph= I2/sqrt(3) = 19.78 A
$P2=3\cdot R2\cdot|I2ph|^2$ = 17606.2 W
$Q2=3\cdot X2\cdot |I2ph|^2$ = -14084 var
Pf2= 0.78 cap

The pf load 1+load 2 can be calculated with the total power, the equivalent impedance or the angle between $\underline{I}a$ and $\underline{U}a'n$

pf load1 and load 2 = 0.84 cap

e) The Wattmeter is measuring the reactive power of the system divided by sqrt(3)

$W=Ib\cdot Uac\cdot\cos(\underline{U}ab\ \underline{I}c) = IL\cdot ULg\cdot \cos(90-\varphi) = IL\cdot ULg\cdot \sin(fi) = QT/\text{sqrt}(3)$ =

=(Q1+Q2+QDL)/sqrt(3) = - 9107.43/sqrt(3) = -5249.67 W

$Q_{DL}= 3\cdot X_{DL}\cdot Ia^2$ =3983.62 var

Alternatively:

$\varphi= \varphi_{\underline{U}an} - \varphi_{\underline{I}a}$= 17.22 – 33.07 = -15.85

W= 527.88 · 36.44 · cos (90+15.85) = -5253.72 W

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