## Electrical Power Engineering Fundamentals

## Third partial exam (16 ${ }^{\text {th }}$ December 2019)

## Note: Please read the questions carefully. All the information you need is written

 below. No questions will be answered related with the exam solutionTwo balanced three phase loads are connected in parallel:
Load 1: $Y$ connected, S1= 3000 VA , pf1 $=0.4$ ind
Load 2: $\Delta$ connected S2= $1500 \mathrm{VA}, \mathrm{pf} 2=0.8 \mathrm{cap}$
The loads are connected to a three-phase generator by means of a transmission line with impedance $3+9 \mathrm{j} \Omega$. The magnitude of the line voltage at the load end of the line $\left(A^{\prime} B^{\prime} C^{\prime}\right)$ is 400 V . The system is positive sequence and the frequency is 50 Hz .

## Considering Ua' $n^{\prime}$ as phase origin calculate:

a) Phasors line current $\underline{\mathbf{I}}_{\underline{a},}, \underline{\mathbf{I}_{\mathbf{a} 1}}, \underline{\mathbf{I}_{\mathrm{a} 2}}(1.5 \mathrm{pt})$
b) Impedance per phase of load 1 and load 2. (1.5 pt)
c) Draw the one phase equivalent of the system (1 pt)
d) Phasor $\underline{U}_{B C}$ and modulus of the voltage drop across the transmission line (1.5 pt)
e) Explain how could the reactive power of the two loads be measured with one Wattmeter. Draw a diagram of the system indicating the point of the circuit where you need to connect the Wattmeter and the connections of it. Demonstrate your answer with a phasor diagram. Calculate the reading of the Wattmeter ( 1.5 pt )
f) Calculate the active power absorbed in the transmission line and the complex power delivered by the generator(1.5 pt)
g) Explain what element or elements should be added to the system and where should they be connected to minimize the complex power absorbed by the transmission line. Calculate the complex power absorbed by the line in that case (The line voltage at $A^{\prime} B^{\prime} C^{\prime}$ remains constant) (1.5 pt)


## SOLUTION

a) Load 1
la1=S1/sqrt(3) $\cdot \mathrm{UL}=3000 / 400 \cdot \operatorname{sqrt}(3)=4.33 \mathrm{~A}$
$\underline{\text { la1 }}=4.33 /-\operatorname{arcos} 0.4=4.33 /-66.42 \mathrm{~A}$
Load 2
$\mathrm{la} 2=\mathrm{S} 2 / \mathrm{sqrt}(3) \cdot \mathrm{UL}=1500 / 400 \cdot \mathrm{sqrt}(3)=2.165 \mathrm{~A}$
$\underline{\mathbf{l a} 2}=2.165 \angle \operatorname{arcos} 0.8=2.165 \angle 36.87 \mathrm{~A}$
$\mathbf{l a T}=\underline{\mathbf{l a}} \underline{\underline{l a 2}}=3.46-2.67 \mathrm{j}=4.37 / 37.65 \mathrm{~A}$
b) $\mathrm{Z1Y}=\underline{\mathrm{U}}_{A^{\prime} \mathbf{N}^{\prime}} / \underline{\mathbf{I a} 1}=21.33+\mathrm{j} 48.88=52.85 / 66.42 \Omega$
$\mathrm{Z} 2 \mathrm{Y}=\underline{\mathbf{U}_{A^{\prime}}} \mathbf{N}^{\prime} / \underline{\mathbf{l a} \mathbf{2}}=83.32-64.11 \mathrm{j}=106.72 /-36.92 \Omega$
$\mathrm{Z} 2 \Delta=3 \cdot \mathrm{Z} 2 \mathrm{Y}=255.95-192.33 \mathrm{j}=320.16 /-36.92 \Omega$
c) The one-phase equivalent includes the Z2Y in parallel with Z1Y
d) $\underline{U}_{A N}=Z_{T i} \cdot \underline{l}_{a T}+\underline{U}_{A^{\prime} N^{\prime}}=266.36 \angle 4.98 \mathrm{~V}$
$\underline{U}_{B C}=\operatorname{sqrt}(3) \cdot 266.36 / 4.98+30-120 \mathrm{~V}$
Voltage drop across the line
$\underline{U}_{A_{A^{\prime}}}=Z_{T \cdot} \cdot \underline{I_{\text {aT }}}=41.46 \angle 33.94 \mathrm{~V}=>$ Voltage drop $=41.46 \mathrm{~V}$
e) The wattmeter should be connected before the two loads, at $A^{\prime} B^{\prime} C^{\prime}$ measuring the current flowing through line and the voltage drop between $\mathrm{B}^{\prime}$ and $\mathrm{C}^{\prime}$. In this situation he Wattmeter measures:
$\mathrm{W}=\mathrm{UL} \cdot \mathrm{laT} \cdot \cos (90-\varphi)=\mathrm{UL} \cdot \mathrm{laT} \cdot \sin (\varphi)=(\mathrm{Q} 1+\mathrm{Q} 2) / \operatorname{sqrt}(3)$
Q1 $=\mathrm{S} 1 \cdot \sin \left(\varphi_{1}\right)=2791.5 \mathrm{var}$
Q2 $=-\mathrm{S} 2 \cdot \sin \left(\varphi_{2}\right)=-900 \mathrm{var}$
$\mathrm{W}=1849.5 / \operatorname{sqrt}(3)=1067.8 \mathrm{~W}$
f) $P_{T L}=3 \cdot R_{T L} \cdot I_{T}{ }^{2}=171.90 \mathrm{~W}$
( $Q_{T L}=3 \cdot X_{T L} \cdot I_{\mathrm{aT}}{ }^{2}=515.71 \mathrm{~W}$
$\mathrm{Sg}=3 \cdot \underline{\mathrm{U}}_{\mathrm{AN}} \cdot \underline{l a T}^{*}=2569.1+2365.5 \mathrm{j}=3492.3 \angle 42.64 \mathrm{VA}$
g) We have to compensate the reactive power of load 1 and load 2 so the total line current drops and the losses in the transmission line drop too. To do so we connect a battery of capacitors in $\Delta$ before the loads, in $A^{\prime}, B^{\prime}, C^{\prime}$.
$Q_{c}=-(Q 1+Q 2)=-1849.5$
$Q_{c \Delta}=-3 \cdot \omega \cdot C_{\Delta} \cdot U_{L}{ }^{2}$
$C_{\Delta}=-Q_{c \Delta} / 3 \cdot \omega \cdot U_{L}{ }^{2}=1849.5 / 3 \cdot 2 \cdot \pi \cdot 400^{2}=1.23 \cdot 10^{-5} \mathrm{~F}$
$\mathrm{I}_{\mathrm{L}}{ }^{\prime}=\mathrm{P}^{\prime} /$ sqrt (3) $\cdot \mathrm{U}_{\mathrm{L}}=2400 /$ sqrt(3) $\cdot 400=3.46 \mathrm{~A}$
$\mathrm{P}^{\prime}=\mathrm{P}=\mathrm{S} 1 \cdot \mathrm{pf} 1+\mathrm{S} 2 \cdot \mathrm{pf} 2=1200+1200=2400 \mathrm{~W}$
$P_{T L}{ }^{\prime}=3 \cdot R_{T L} \cdot I_{L}{ }^{\prime 2}=107.99 \mathrm{~W}$
$Q_{t L^{\prime}}=3 \cdot X L \cdot I^{\prime} 2=323.98 \mathrm{var}$
$\mathrm{S}_{\text {т }}{ }^{\prime}=107.99+\mathrm{j} 323.98 \mathrm{VA}$

