Module 1: Basic concepts

Electrical power engineering fundamentals
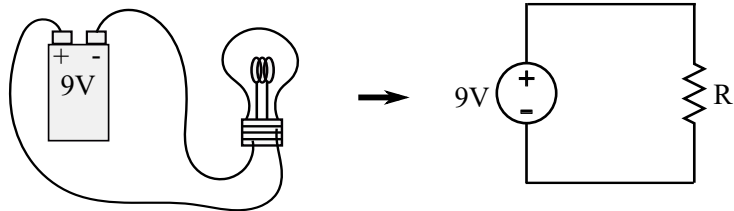
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Circuit models

Real systems can be modelled by means of electrical circuits:

Circuit analysis rules facilitate the calculation of currents, voltages and powers at different parts of the systems.

Assumptions

- **Lumped-elements circuits**: each element is concentrated in one point of the space. Currents move instantaneously throughout the circuit.

- **Steady state circuits**: the circuit has remained under the same conditions for enough time to reach permanent regime.

Analysis of electrical circuits

- ▶ Electrical circuit: Excitation / Response
- ▶ Apply basic rules to determine the circuit response as a function of the excitation.
- ▶ Laws are based in the principles of electromagnetism.

Charge

▶ Property of the materials

▶ Origin of electrostatic interaction.

▶ Bipolar nature

▶ Interaction

$$\longleftarrow\oplus \quad \oplus\longrightarrow \qquad \oplus\rightarrow \ \leftarrow\ominus \qquad \longleftarrow\ominus \quad \ominus\longrightarrow$$
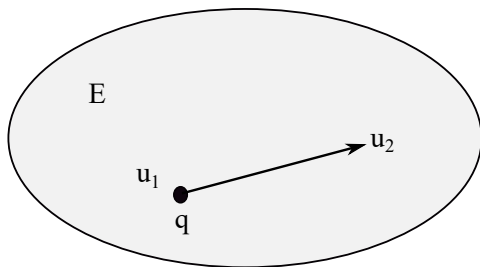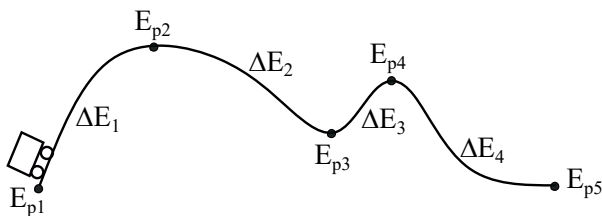
▶ Unit SI: Coulombs [C]

$$q_{e^-} = -1.6 \cdot 10^{-19} C$$

Voltage

- The presence of charge originates an electric field distribution
- A charge within an electric field possesses an **electric potential energy** which is the so called potential or **voltage** at this point.

Gravitational simile

$$E_{pk} = m \cdot g \cdot h_k$$

At each point of the path the trolley has a certain potential energy and when the trolley moves between two points it losses or gain potential energy
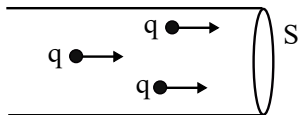
Voltage difference

The **voltage difference** between two points of the space is the work that must be supplied to move a charge between these two points.

$$u = \frac{dw}{dq}$$

Voltage is measured in Volts [V] in the SI.

Current

Free electrons can move in conductive materials if they are subjected to a certain voltage difference.


Electric **current** or intensity is defined as the total amount of electric charge that flows through the section of a conductive material per unit of time.
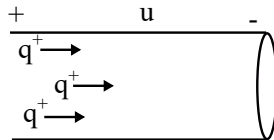
$$i(t) = \frac{dq}{dt}$$

Current is measured in Amperes [A] in the SI.

Sign criteria

▶ **Electronic current criteria**. Negative charges move from lower voltages to higher voltages. What happens in real systems.

▶ **Conventional current criteria**. Positive charges move from higher voltages to lower voltages. We will adopt this criteria.
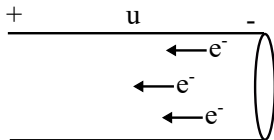
Electronic current criteria

Conventional current criteria

Electric power

is defined as work done per unit of time

$$p = \frac{dw}{dt}$$

Electric power can be calculated as:

$$p = \frac{dw}{dt} = \frac{u \cdot dq}{dt} = u \cdot i$$

Electric power is measured in Watts [W] in the SI

Power absorbed or delivered

▶ An element absorbs power when current moves from a point of higher voltage towards a point at lower voltage. The charges loose energy when going through this element.

▶ An element delivers power when current flows from a point at lower voltage towards a point with higher voltage. Then, the charges gain energy on their way through the element.

absorbs power   delivers power
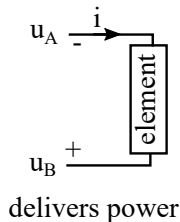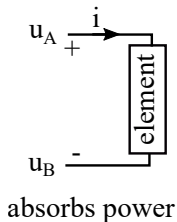
Sign criteria

In electric circuits there is always a power balance between the power delivered by sources and the power absorbed by resistors.

The sign criteria that we will adopt is:

▶ Power delivered by sources is taken as positive and power absorbed by sources is taken as negative.

▶ Power absorbed by resistors is taken as positive (resistors never deliver energy)

**Power balance**

$$p_{sources} = p_{resistors}$$

Kirchhoff's current law

Kirchhoff's current law (KCL), also referred to as first Kirchhoff's law, is based in the charge conservation principle and states that **the algebraic sum of the current at any node of a circuit equals zero**.

$$\sum i = 0$$

Kirchhoff's current law

Electric charges flow into the node or out of the node through the branches connected to it, but no current is stored in the junction.
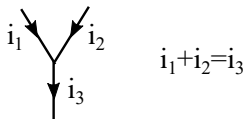

$$i_1 + i_2 = i_3$$

The adoption of the sign criteria is arbitrary, but we will take:

Currents flowing out of the node $+$

Currents flowing into the node -

$$\sum i = 0 \quad => \quad -i_1 - i_2 + i_3 = 0$$

Example

In the following circuit, calculate the value of i_R
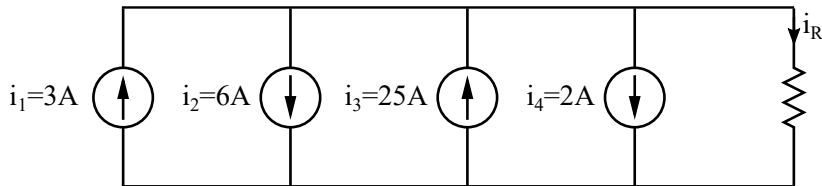
Applying KCL we get:

$$\sum i = 0 \quad => \quad -i_1 + i_2 - i_3 + i_4 + i_R = 0$$
$$i_R = 3 - 6 + 25 - 2 = 20A$$

Kirchhoff's voltage law

Kirchhoff's voltage law (KVL), also referred to as second Kirchhoff's law, is based in the energy conservation principle and states that **the algebraic sum of the voltages around a closed path in a circuit equals zero**.

$$\sum u = 0$$

Kirchhoff's voltage law

Charges moving around a closed path loose energy at some sections of the path and gain energy at some others but the total gain zero.
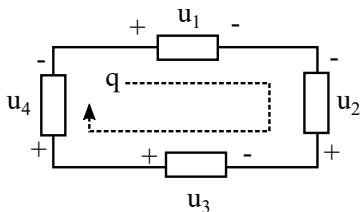
$u_1 + u_4 = u_2 + u_3$

We will take:

Voltage drops $+$

Voltage rises -

$$\sum u = 0 \quad => \quad u_1 - u_2 - u_3 + u_4 = 0$$

Mechanical simile

In a roller-coaster where the speed of the trolley is the same at the beginning and at the end of the path, the potential energy rises at growing sections equals the energy drops at decreasing sections.
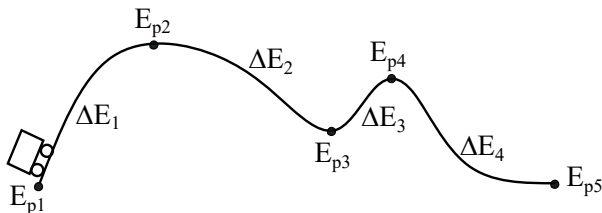
$$\Delta E_1 = E_{p2} - E_{p1}, \ \Delta E_2 = E_{p3} - E_{p2}, \ \Delta E_3 = E_{p4} - E_{p3}, \ \Delta E_4 = E_{p5} - E_{p4}$$

$$\Delta E_1 + \Delta E_3 = \Delta E_2 + \Delta E_4$$

Example

Calculate the value of u_5:
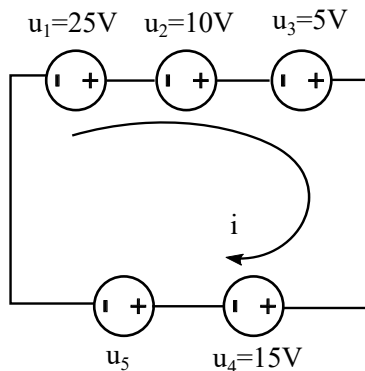
$$\sum u = 0 \quad => \quad -u_1 - u_2 - u_3 + u_4 + u_5 = 0$$

$$u_5 = 25 + 10 + 5 - 15 = 25V$$

Active and passive elements

▶ Active elements: Supply energy to the circuit

Battery $\quad +\quad$ Generator $\quad$ (G)
$\quad\quad -$
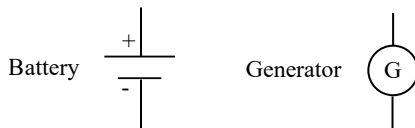
▶ Passive elements: Absorb or store electric energy.

Resistor   Inductor   Capacitor
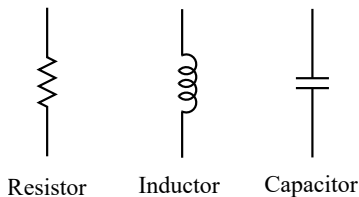
Resistors

▶ Resistors are used to model elements that transform electric energy into heat (i.e. bulbs, electric radiators..).

▶ For circuit analysis we characterize these elements by means of the so called **electrical resistance or resistance** (R).

▶ Resistance is measured in Ohms [Ω] in the SI.
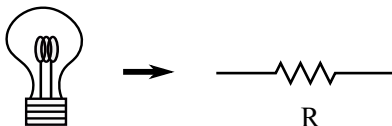
Physical meaning

- ▶ The concept of electrical resistance is related with the loss of energy experienced by the charges flowing though a certain section of a conductive material.

- ▶ The charges collide and those collisions dissipate energy which is transformed into heat.

- ▶ The electric resistance of an element depends on the nature of the material it is constituted of and on its geometry.
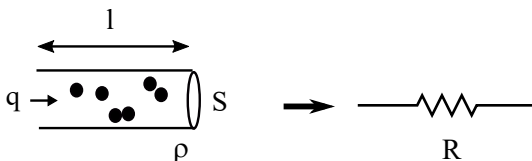
Resistivity

- ▶ The resistivity (ρ) of a material is a physical property that is related with how easily electrons move inside it. ▶ Link

- ▶ Resistivity is the inverse of conductivity (σ) and is measured in $[\Omega] \cdot [m]^{-1}$ in the SI.

$$R = \frac{\rho \cdot l}{S}$$

- ▶ Additionally, we define the conductance of an element (G) as the inverse of its resistance. Conductance is measured in Siemens [S] in the SI.

$$G = \frac{1}{R}$$

Ohms's law

- ▶ Electric charges always lose energy when they flow though a resistor.

- ▶ In circuit analysis the loss of energy is characterized as a voltage drop. **The relative polarity between voltage and current in a resistor is always as shown below**:

- ▶ **Ohm's law** provides the relation between the current and the voltage drop across a resistor:

$$u = R \cdot i$$

Power absorbed by a resistor

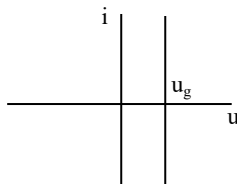Resistors **always** absorb energy (i.e. never deliver it).

$$p_R = u \cdot i$$
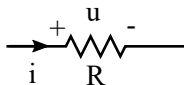
$$u = R \cdot i$$

Then:

$$p_R = R \cdot i^2$$

$$p_R = \frac{u^2}{R}$$

Ideal voltage sources

▶ Ideal voltage sources keep a constant voltage drop across their terminals regardless the current flowing through them.
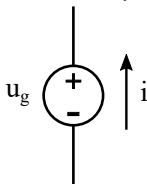
▶ Ideal voltage sources provide DC or AC voltage.
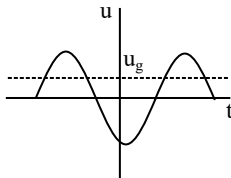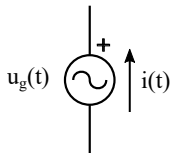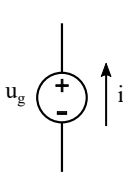
Ideal voltage sources

To characterize an ideal DC voltage source we need to know its
output voltage, which is the voltage drop across its terminals,
and its **polarity**.
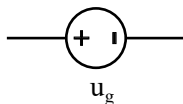
The power supplied by a voltage source can be calculated as the
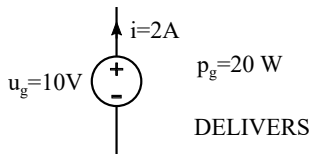product of the output voltage and the current flowing through it.


$$p_g = u_g \cdot i$$

Ideal voltage sources

▶ Although voltage sources are active elements they might absorb power.

▶ A source delivers power if current flows from $+$ to - and delivers power if current flows from - to $+$

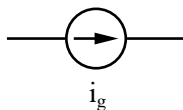▶ Sign criteria: Power delivered by sources $+$. Power absorbed by sources -

Example
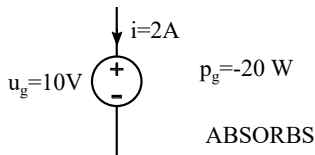
Ideal current sources

- Ideal current sources keep a constant current flow through them regardless the voltage drop across their terminals.
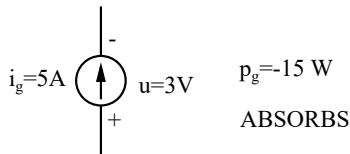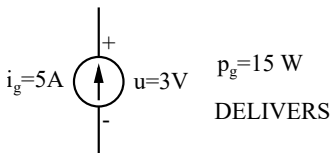
- Ideal current sources can be DC or AC.
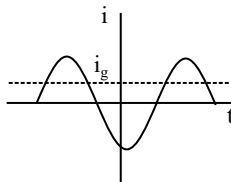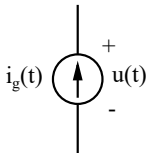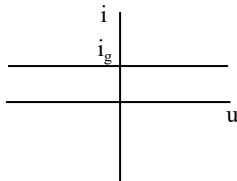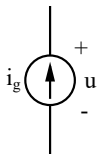
Ideal current sources

▶ To define an ideal current source we need to know its **output current**, and its **polarity** (indicated by the direction of the arrow).

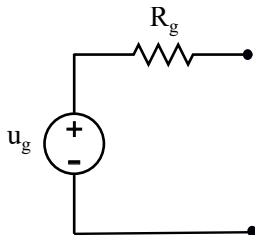$$i_g$$

▶ Power supplied by an ideal current source:

$$p_g = i_g \cdot u$$

Example

i_g=5A u=3V p_g=15 W   DELIVERS

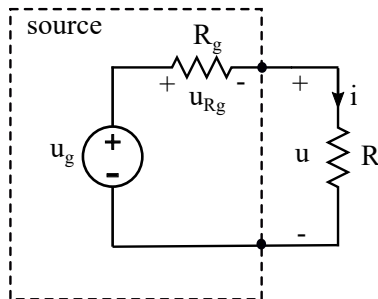i_g=5A u=3V p_g=-15 W   ABSORBS

Real voltage sources

A real voltage source is modelled as an ideal voltage source in series with a resistor.

The supplied voltage is not constant but depends on the current flowing through the circuit
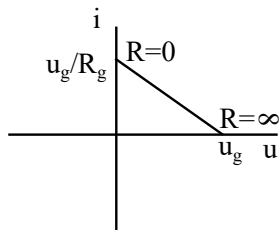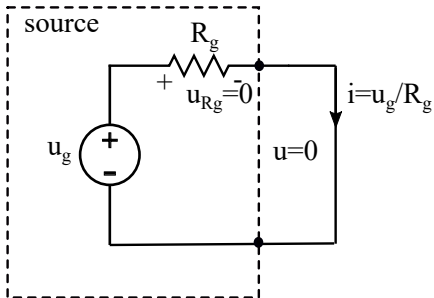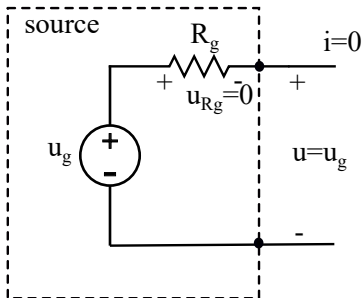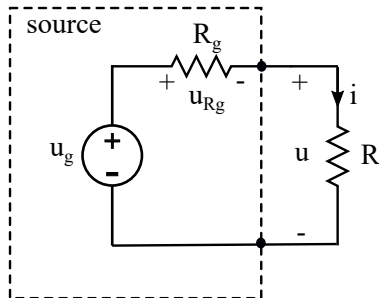
Real voltage sources

$$u = u_g - R_g \cdot i$$

where

$$i = \frac{u_g}{R_g + R}$$

Real voltage sources

source

R_g

$i=0$

$+$
$u_{Rg}=0$ $+$

u_g $+$ $-$

$u=u_g$

$-$

source

R_g

$+$
$u_{Rg}=0$

u_g $+$ $-$

$i=u_g/R_g$

$u=0$

i

u_g/R_g $R=0$

$R=\infty$

u_g u

Efficiency

Power supplied to the resistor:

$$p_g = R \cdot i^2 = R \cdot \frac{u_g^2}{(R_g + R)^2}$$

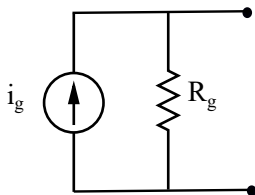Internal losses of the source:

$$p_{R_g} = u_{R_g} \cdot i = R_g \cdot \frac{u_g^2}{(R_g + R)^2}$$
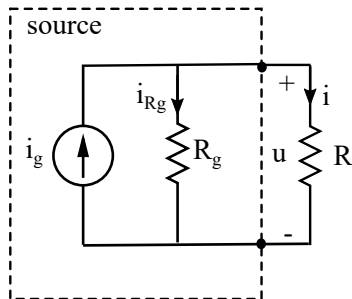
Efficiency of the source

$$\eta = \frac{p_g}{p_g + p_{R_g}} = \frac{R}{R_g + R}$$

Real current source

A real current sources is modelled as an ideal current source in parallel with a resistor.

Real current source

The supplied current (i) is not constant but depends on the voltage drop (u) across its terminals.

$$i = i_g - \frac{u}{R_g}$$

Circuit analysis

To tackle the analysis of resistive electrical circuits we will use the basic rules introduced so far:
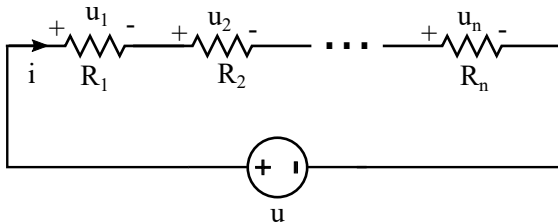
▶ Kirchhoff laws

$$\sum i = 0 \qquad\qquad \sum u = 0$$

▶ Ohm's law

$$u = R \cdot i$$

Additionally it is helpful to simplify electrical circuits by associating elements or transforming them into equivalent ones.
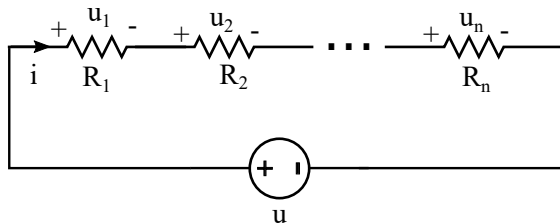
Series connected resistors

Two or more elements are connected in **series** if the **same current** flows through them.

Series connected resistors

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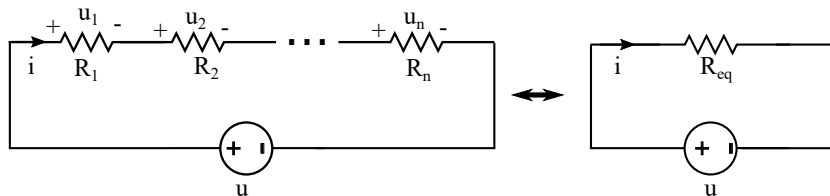

$$u_1 = R_1 \cdot i \quad u_2 = R_2 \cdot i \quad ... \quad u_n = R_n \cdot i$$

$$u = u_1 + u_2 + ... + u_n = R_1 \cdot i + R_2 \cdot i + ... + R_n \cdot i = (R_1 + R_2 + ... + R_n) \cdot i$$

Series connected resistors

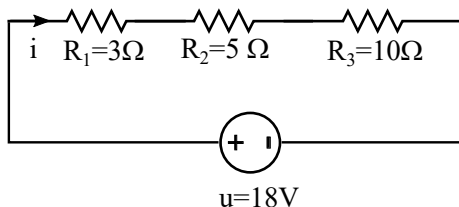The set of n resistors can be redrawn as an equivalent resistance R_{eq}:

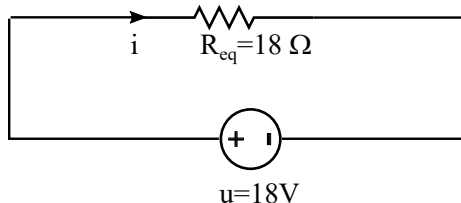$$R_{eq} = R_1 + R_2 + ... + R_n = \sum_k R_k$$
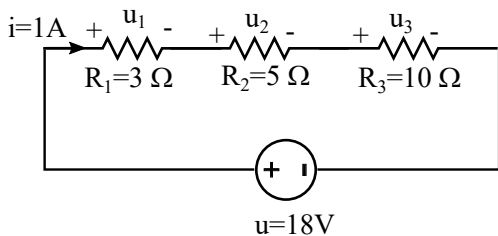
$$u = R_{eq} \cdot i$$

Example

Calculate the current and voltage drop across R_1, R_2 and R_3.

i $R_1 = 3\Omega$ $R_2 = 5\ \Omega$ $R_3 = 10\Omega$
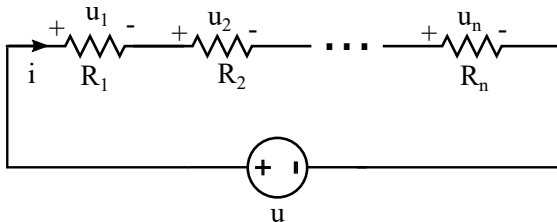
$u = 18V$

Example

$$i = \frac{u}{R_{eq}} = \frac{18}{18} = 1A$$

$$u_1 = R_1 \cdot i = 3V \quad u_2 = R_2 \cdot i = 5V \quad u_3 = R_3 \cdot i = 10V$$

The voltage divider equation

Some electronic devices uses nets formed by several resistors to obtain a fraction of the output voltage of a source. The voltage drop across each of the series connected resistors is a portion of the voltage of the source.
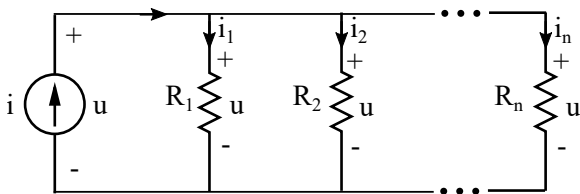

The equation to calculate the voltage drop across resistor k is:

$$u_k = \frac{R_k}{R_{eq}} \cdot u$$

Parallel connected resistors

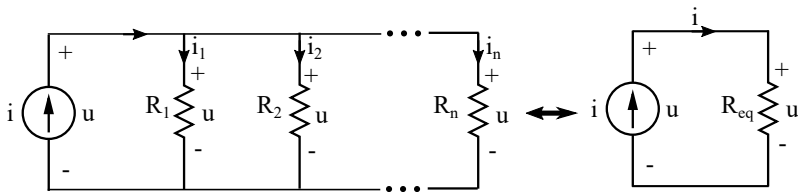Two or more elements are connected in **parallel** if they have the **same voltage drop across them**.

$$i_1 = \frac{u}{R_1} \qquad i_2 = \frac{u}{R_2} \qquad ... \qquad i_n = \frac{u}{R_n} = u \cdot G_n$$

$$i = i_1 + i_2 + ... + i_n = \frac{u}{R_1} + \frac{u}{R_2} + ... + \frac{u}{R_n} = u \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}\right)$$

Parallel connected resistors

The set of n resistors can be redrawn as an equivalent resistance R_{eq}:

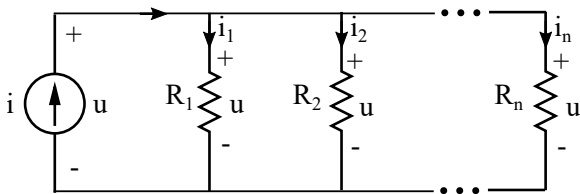
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n} = \sum_k \frac{1}{R_k}$$

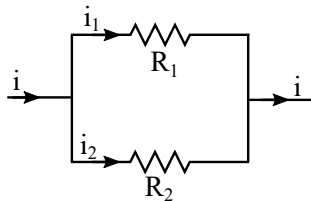In terms of conductance:

$$G_{eq} = \sum_k G_k$$

Current divider

A current divider consists of a circuit constituted by several parallel connected resistors that can be used to obtain a fraction of the output current of a source

The current at the resistor k is

$$i_k = \frac{G_k}{G_{eq}} \cdot i$$

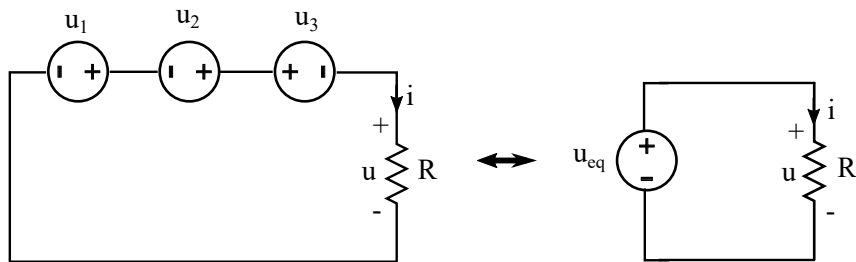# Current divider for two resistors in parallel

$$i_1 = \frac{R_2}{R_1 + R_2} \cdot i \qquad\qquad i_2 = \frac{R_1}{R_1 + R_2} \cdot i$$

Ideal voltage sources in series

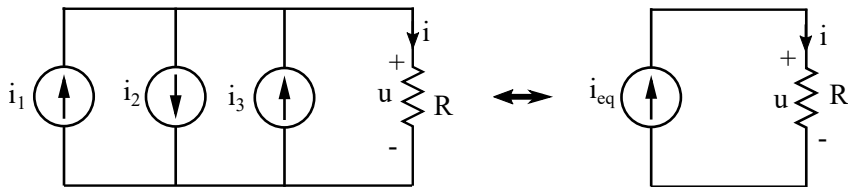Ideal voltage sources in series can be replaced by an equivalent voltage source.

The output voltage of the equivalent voltage source is the sum of the voltages of the individual sources taking into account their polarity

$$u_{eq} = u_1 + u_2 - u_3$$

Ideal current sources in parallel

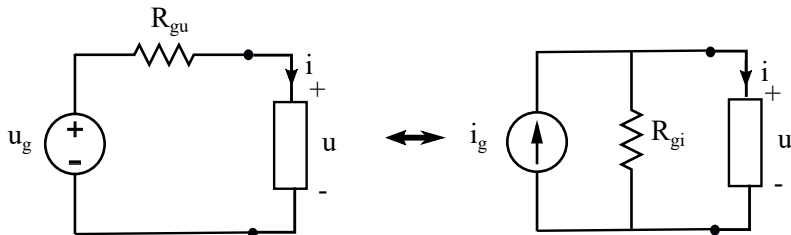Ideal current sources in parallel can be replaced by an equivalent current source.

The output current of the equivalent ideal current source is the algebraic sum of the output currents of the ideal current sources:

$$i_{eq} = i_1 - i_2 + i_3$$

Transformation of real sources

Real voltage sources can be redrawn as equivalent real current sources and the other way around.

$$u = u_g - R_{gu} \cdot i \qquad\qquad i = i_g - \frac{u}{R_{gi}}$$

The sources are equivalent if:

$$i_g = \frac{u_g}{R_{gu}} \qquad and \qquad R_{gu} = R_{gi}$$

Inductors and capacitors

▶ We have studied tat resistors are passive elements that transform electric energy into heat

▶ Now we introduce another type of passive elements which, unlike resistors, do not dissipate energy but store it instead

▶ Capacitors are elements that store electric energy in an electric field

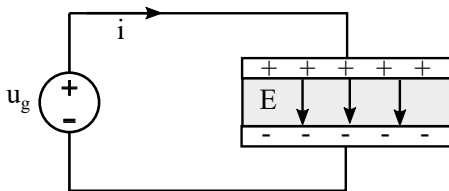▶ Inductors are elements that store electric energy in a magnetic field

Capacitor

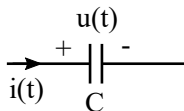Capacitor: two metallic plates separated a distance d with a dielectric material between them.
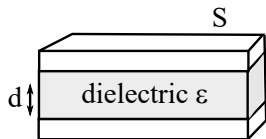
$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$$

Capacitance is measured in Farads [F] in the SI.

$$q = C \cdot u \qquad C = \frac{\varepsilon \cdot S}{d}$$

Capacitor: relation between voltage and current

$$u(t)$$

$$\xrightarrow{\;}\; \overset{+}{}\; {\Large ||}\; \overset{-}{}$$

i(t)   C

$$q = C \cdot u \;\; => \;\; \frac{dq}{dt} = C \cdot \frac{du}{dt} => \;\; i(t) = C \cdot \frac{du(t)}{dt}$$

Voltage drop across the terminals of the capacitor for a certain current:

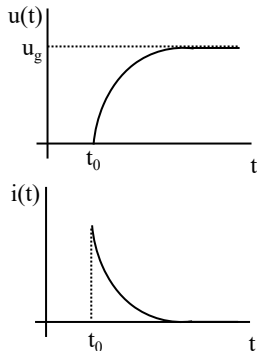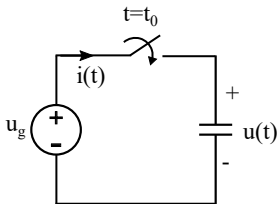$$u(t) = u(t_0) + \frac{1}{C} \int_{t_0}^{t} i(t) dt$$

The voltage across a capacitor always varies smoothly.
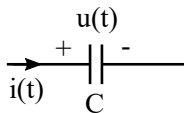
Capacitor in DC

In DC circuits capacitors behave as open circuit in **steady state**:

$$i(t) = C \cdot \frac{du(t)}{dt} = C \cdot \frac{du_g}{dt} = 0$$

In **transient conditions**, the voltage changes and the current is not zero.
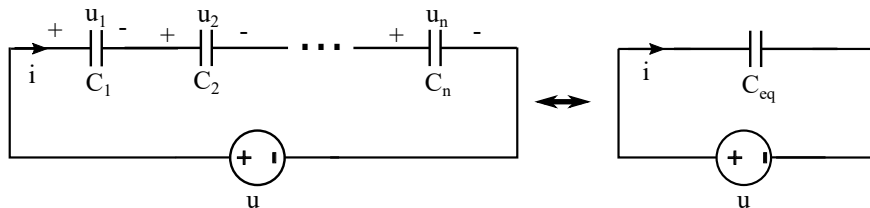
Power and energy stored in a capacitor

Power:

$$p = u \cdot i = u \cdot C \cdot \frac{du}{dt}$$

Energy:

$$p = \frac{dw}{dt} \quad => \quad \int dw = C \int u \cdot du$$

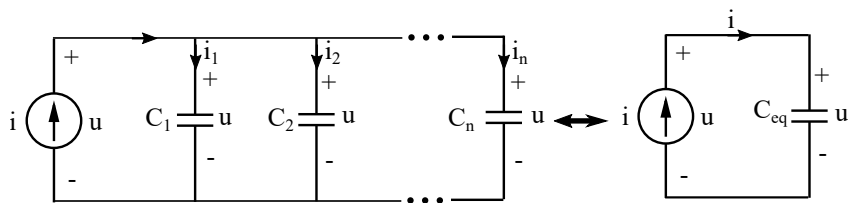$$w = \frac{1}{2} \cdot C \cdot u^2$$

Association of capacitors in series

$$u = u_1 + u_2 + ... + u_n$$

$$\frac{du}{dt} = \frac{du_1}{dt} + \frac{du_2}{dt} + ... + \frac{du_n}{dt} = \left(\frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}\right) \cdot i = \frac{1}{C_{eq}} \cdot i$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n} = \sum_k \frac{1}{C_k}$$
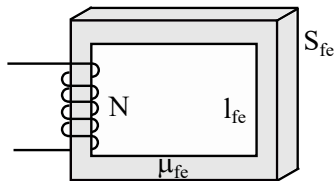
Association of capacitors in parallel

$$i = i_1 + i_2 + ... + i_n = (C_1 + C_2 + ... + C_n) \cdot \frac{du(t)}{dt} = C_{eq} \cdot \frac{du(t)}{dt}$$

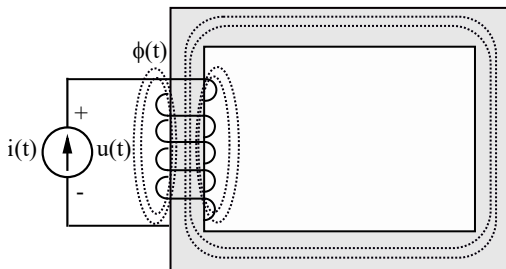$$C_{eq} = C_1 + C_2 + ... + C_n = \sum_k C_k$$

Inductors

Inductors are passive elements that take energy from a source and store it in a magnetic field.
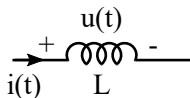

Inductance:

$$L = \frac{N^2 \cdot S_{fe} \cdot \mu_{fe}}{l_{fe}} \quad [H]$$

$$N \cdot \phi = L \cdot i$$

Inductors: relation between voltage and current

Faraday law:

$$u = N \cdot \frac{d\phi}{dt}$$

$$N \cdot \phi = L \cdot i \quad => \quad N \cdot \frac{d\phi}{dt} = L \cdot \frac{di}{dt}$$

$$u(t) = L \cdot \frac{di(t)}{dt} \quad i(t) = i(t_0) + \frac{1}{L} \int_{t_0}^{t} u(t)dt$$

Current through an inductor always varies smoothly.

Inductors in DC circuits

If an inductor is fed with a DC current source of constant value i_g, the voltage across the inductor would be zero:

$$u(t) = L \cdot \frac{di(t)}{dt} = L \cdot \frac{di_g}{dt} = 0$$

In DC, in steady state, **an inductor behaves as a short circuit**.

In transient conditions the current flowing through the inductor changes and the voltage drop across the inductor is not null until steady state is reached.
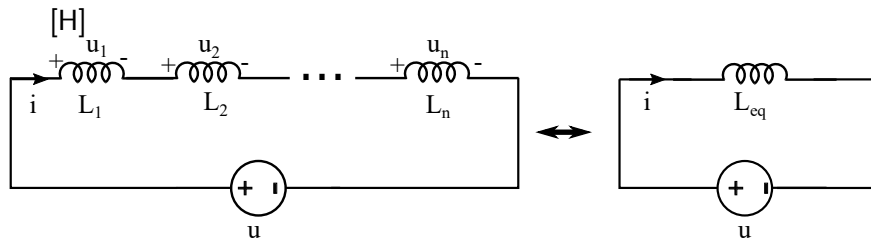
Power and energy

Power absorbed by an inductor

$$p = u \cdot i = i \cdot L \frac{di}{dt}$$

Energy stored in a inductor:

$$p = \frac{dw}{dt} = L \cdot i \cdot \frac{di}{dt} \quad => \quad \int dw = L \int i \cdot di$$
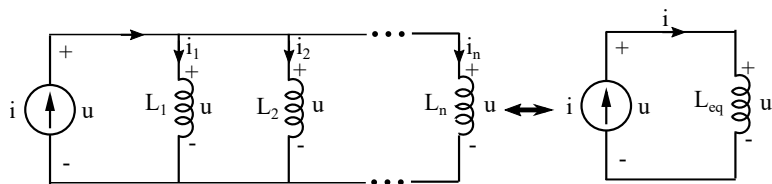
$$w = \frac{1}{2} \cdot L \cdot i^2$$

Association of inductors in series

$$u = u_1 + u_2 + ... + u_n = (L_1 + L_2 + ... + L_n) \cdot \frac{di}{dt} = L_{eq} \cdot \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + ... + L_n = \sum_i L_i$$
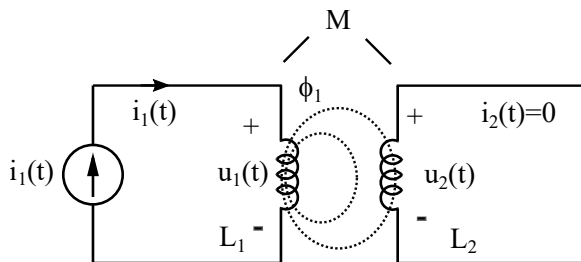
Association of inductors in parallel

$$i = i_1 + i_2 + ... + i_n$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + ... + \frac{di_n}{dt} = \left(\frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_n}\right) \cdot u = \frac{1}{L_{eq}} \cdot u$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_n} = \sum_i \frac{1}{L_i}$$
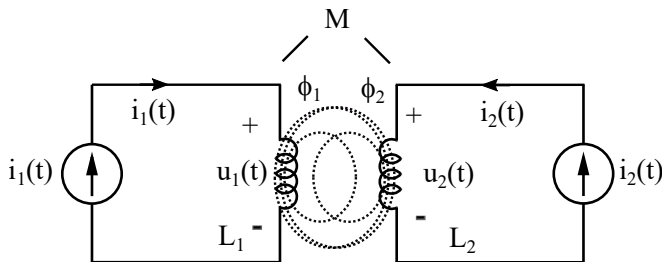
Coupled inductors

Mutual inductance coefficient (M): Degree of coupling between the inductors. It is measured in [H] in the SI.

$$u_1(t) = L_1 \cdot \frac{di_1(t)}{dt} \qquad u_2(t) = M \cdot \frac{di_1(t)}{dt}$$
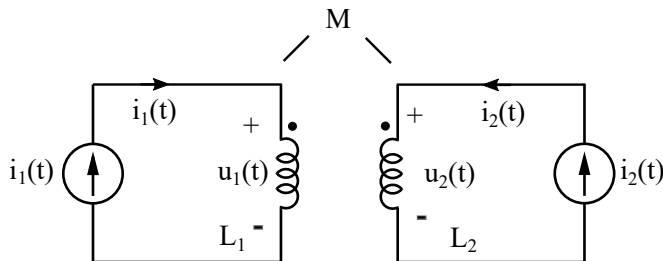
Coupled inductors

If inductor 2 is supplied with current i_2 a second flux ϕ_2 is be created:

$$u_1(t) = L_1 \cdot \frac{di_1(t)}{dt} + M \cdot \frac{di_2(t)}{dt}$$

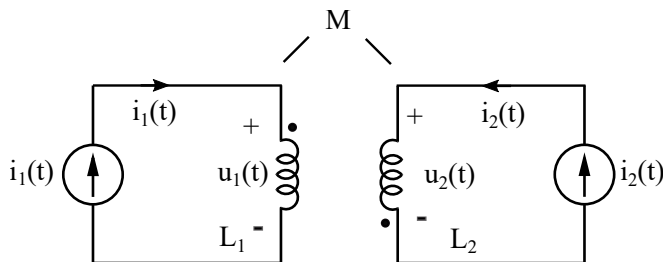$$u_2(t) = L_2 \cdot \frac{di_2(t)}{dt} + M \cdot \frac{di_1(t)}{dt}$$

Polarity of the coupling: dot convention

$$u_1(t) = L_1 \cdot \frac{di_1(t)}{dt} + M \cdot \frac{di_2(t)}{dt}$$

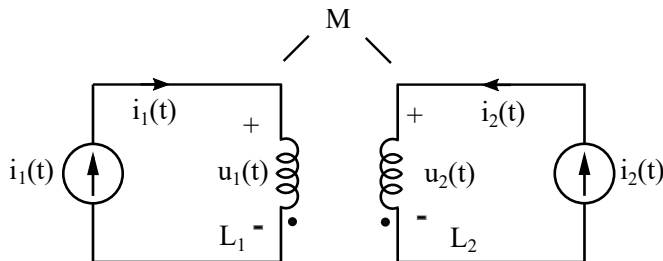$$u_2(t) = L_2 \cdot \frac{di_2(t)}{dt} + M \cdot \frac{di_1(t)}{dt}$$

Polarity of the coupling

$$u_1(t) = L_1 \cdot \frac{di_1(t)}{dt} - M \cdot \frac{di_2(t)}{dt}$$

$$u_2(t) = L_2 \cdot \frac{di_2(t)}{dt} - M \cdot \frac{di_1(t)}{dt}$$
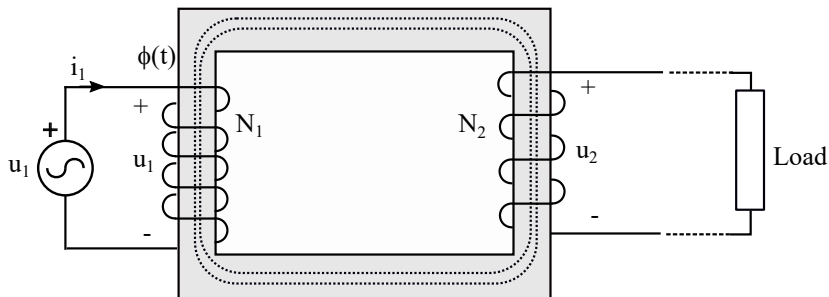
Polarity of the coupling

$$u_1(t) = L_1 \cdot \frac{di_1(t)}{dt} + M \cdot \frac{di_2(t)}{dt}$$

$$u_2(t) = L_2 \cdot \frac{di_2(t)}{dt} + M \cdot \frac{di_1(t)}{dt}$$

Applications of coupled inductors: Transformers

Transformers are used to change the voltage level of electric energy.

If the number of coils of the two inductors is different, the voltage u_2 differs from u_1. Transformation relation:

$$r_t = \frac{N_1}{N_2} = \frac{u_1}{u_2}$$