

Module 4: Analysis of three-phase systems Electrical power engineering fundamentals

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Configuration of three-phase systems

- Most power systems are three-phase.
- Each part of the system is called **phase**



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Three phase generators

Three phase generators generate three sinusoidal voltages of the same amplitude and phase shift 120^{o}



$$u_{a}(t) = \sqrt{2} \cdot U \cdot \cos(\omega t)$$
$$u_{b}(t) = \sqrt{2} \cdot U \cdot \cos(\omega t - 120^{\circ})$$
$$u_{c}(t) = \sqrt{2} \cdot U \cdot \cos(\omega t + 120^{\circ})$$

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Generators in wye and delta

We represent three-phase generators as three AC sources connected in $\ensuremath{\textbf{wye}}$ or $\ensuremath{\textbf{delta}}$



Positive and negative phase sequence





Positive sequence

 $\underline{\mathbf{U}_{a}} = U \angle 0^{\circ}$ $\underline{\mathbf{U}_{b}} = U \angle -120^{\circ}$ $\underline{\mathbf{U}_{c}} = U \angle 120^{\circ}$

Negative sequence

- $\underline{\mathbf{U}_a}=\textit{U}\angle \texttt{0}^o$
- $\underline{\mathbf{U}}_{\underline{b}} = U \angle 120^{o}$
- $\underline{\mathbf{U}_{c}} = U \angle -120^{o}$

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Three-phase loads

- Three-phase loads are represented in the frequency domain, as a set of three impedances connected between them.
- We limit our analysis to balanced systems in which the impedance connected to each phase has the same value
- Loads can be connected in wye or delta



Balanced wye-wye system systems

In wye wye systems the neutral points of the generators and the loads might be connected by means of a **neutral wire**.



The current of each phase flows from the generator towards the load and returns through the neutral wire

Currents in a balanced wye-wye system

$$\underline{\mathbf{I}}_{\underline{a}} = \frac{\underline{\mathbf{U}}_{\underline{a}}}{Z_{Y}} = \frac{U \angle 0}{Z_{Y}} \qquad \underline{\mathbf{I}}_{\underline{b}} = \frac{\underline{\mathbf{U}}_{\underline{b}}}{Z_{Y}} = \frac{U \angle -120^{\circ}}{Z_{Y}} \qquad \underline{\mathbf{I}}_{\underline{c}} = \frac{\underline{\mathbf{U}}_{\underline{c}}}{Z_{Y}} = \frac{U \angle 120^{\circ}}{Z_{Y}}$$

The currents also form a three phase system. If $Z_Y = |Z_Y| \angle \theta$



Inductive load

Capacitive load

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Current through the neutral wire

As the current flowing through the neutral is zero the wire is often suppressed.

$$\underline{\mathbf{I}_{N}} = \underline{\mathbf{I}_{a}} + \underline{\mathbf{I}_{b}} + \underline{\mathbf{I}_{c}} = \frac{U}{Z_{Y}} \cdot (1 \angle 0 + 1 \angle -120^{\circ} + 1 \angle 120^{\circ}) = 0$$



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Savings of three-phase system

- Lower material investment
- Lower line losses for the same power transfer

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Line and phase voltages and currents

- Phase-voltage: Voltage drop across a single phase of the generator or the load. The phase-voltage of the generator is the voltage drop across the terminals of one of the ideal voltage sources; the phase voltage of the load is the voltage drop across one of the impedances that constitutes the load.
- Line-votage: Voltage drop between any couple of lines. We could obtain the line voltage at the generator side of the system, or the line voltage at the load side.
- Phase current: Current in a single phase, i.e current flowing through one of the ideal sources or through one of the impedances.

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Line current: Current in a single line

Line and phase voltages and currents



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Relation between the line and phase magnitudes in a wye-wye system

Relation between the line and phase voltages:

$$\underline{\mathbf{I}}_L = \underline{\mathbf{I}}_{Ph}$$

Relation between the line and phase voltages:

$$\underline{\mathbf{U}}_{Pha} = \underline{\mathbf{U}}_{a} \qquad \underline{\mathbf{U}}_{Phb} = \underline{\mathbf{U}}_{b} \qquad \underline{\mathbf{U}}_{Phc} = \underline{\mathbf{U}}_{c}$$

$$\underline{\mathbf{U}}_{La} = \underline{\mathbf{U}}_{a} - \underline{\mathbf{U}}_{b} = U\angle 0 - U\angle -120^{\circ} = \sqrt{3} \cdot U\angle 30^{\circ} = \sqrt{3} \cdot \underline{\mathbf{U}}_{Pha}\angle 30^{\circ}$$
$$\underline{\mathbf{U}}_{Lb} = \underline{\mathbf{U}}_{BC} = \underline{\mathbf{U}}_{b} - \underline{\mathbf{U}}_{c} = \sqrt{3} \cdot \underline{\mathbf{U}}_{Phb}\angle 30^{\circ}$$
$$\underline{\mathbf{U}}_{Lc} = \underline{\mathbf{U}}_{CA} = \underline{\mathbf{U}}_{c} - \underline{\mathbf{U}}_{a} = \sqrt{3} \cdot \underline{\mathbf{U}}_{Phc}\angle 30^{\circ}$$

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Relation between the line and phase magnitudes in a wye-wye system



The line voltages are $\sqrt{3}$ times larger that the phase voltages and lead the phase voltages by 30^o

Relation between the line and phase magnitudes in a delta-delta system



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Relation between the line and phase magnitudes in a delta-delta system

Phase and line voltages:

$$\underline{\mathbf{U}}_{LINE} = \underline{\mathbf{U}}_{PHASE}$$

Phase currents:

$$\underline{\mathbf{I}}_{Pha} = \underline{\mathbf{I}}_{B'C'} = \frac{\underline{\mathbf{U}}_{a}}{Z_{\Delta}} = \frac{U\angle 0}{Z_{\Delta}}$$

$$\underline{\mathbf{I}}_{Phb} = \underline{\mathbf{I}}_{A'B'} = \frac{\underline{\mathbf{U}}_{b}}{Z_{\Delta}} = \frac{U\angle -120^{\circ}}{Z_{\Delta}}$$

$$\underline{\mathbf{I}}_{Phc} = \underline{\mathbf{I}}_{C'A'} = \frac{\underline{\mathbf{U}}_c}{Z_\Delta} = \frac{U\angle 120^\circ}{Z_\Delta}$$

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Relation between the line and phase magnitudes in a delta-delta system

Line currents:

$$\underline{\mathbf{I}}_{La} = \underline{\mathbf{I}}_{Pha} - \underline{\mathbf{I}}_{Phb} = \frac{U\angle 0}{Z_{\Delta}} - \frac{U\angle -120^{\circ}}{Z_{\Delta}} = \sqrt{3} \cdot \frac{U}{Z_{\Delta}} \angle -30^{\circ} = \sqrt{3} \cdot \underline{\mathbf{I}}_{Pha} \angle -30^{\circ}$$
$$\underline{\mathbf{I}}_{Lb} = \sqrt{3} \cdot \underline{\mathbf{I}}_{Phb} \angle -30^{\circ}$$

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$$\underline{\mathbf{I}}_{Lc} = \sqrt{3} \cdot \underline{\mathbf{I}}_{Phc} \angle -30^{o}$$

The line currents are $\sqrt{3}$ times larger that the phase currents and lag the phase currents by 30°

Summary



Delta configuration

$$U_{line} = U_{phase}$$

$$I_{line} = \sqrt{3} \cdot I_{phase}$$
$$\underline{I}_{LA} = \sqrt{3} \cdot \underline{I}_{phA} \angle -30$$

Wye configuration $U_{line} = \sqrt{3} \cdot U_{phase}$ $I_{line} = I_{phase}$ $\underline{\mathbf{U}}_{LA} = \sqrt{3} \cdot \underline{\mathbf{U}}_{phA} \angle 30$ I_L A Iph Z_{Δ} Z_{Δ} U_L U_{ph} В ZΔ • • • • • • •

Analysis of three-phase systems: one-phase equivalent



$$-\underline{\mathbf{U}}_{a} + \underline{\mathbf{I}}_{a} \cdot (Z_{TL} + Z_{Y}) + (\underline{\mathbf{I}}_{a} + \underline{\mathbf{I}}_{b} + \underline{\mathbf{I}}_{c}) \cdot Z_{N} = 0$$

$$-\underline{\mathbf{U}}_{b} + \underline{\mathbf{I}}_{b} \cdot (Z_{TL} + Z_{Y}) + (\underline{\mathbf{I}}_{a} + \underline{\mathbf{I}}_{b} + \underline{\mathbf{I}}_{c}) \cdot Z_{N} = 0$$

$$-\underline{\mathbf{U}}_{c} + \underline{\mathbf{I}}_{c} \cdot (Z_{TL} + Z_{Y}) + (\underline{\mathbf{I}}_{a} + \underline{\mathbf{I}}_{b} + \underline{\mathbf{I}}_{c}) \cdot Z_{N} = 0$$

Analysis of three-phase circuits



As there is no current flow through the neutral wire, the three phases can be analysed as independent circuits

- $-\underline{\mathbf{U}_{a}}+\underline{\mathbf{I}_{a}}\cdot\left(Z_{TL}+Z_{Y}\right)=0$
- $-\underline{\mathbf{U}}_{\underline{b}} + \underline{\mathbf{I}}_{\underline{b}} \cdot (Z_{TL} + Z_{Y}) = 0$ $-\underline{\mathbf{U}}_{\underline{c}} + \underline{\mathbf{I}}_{\underline{c}} \cdot (Z_{TL} + Z_{Y}) = 0$

One phase equivalent of a three-phase circuit

- ▶ We represent the circuit with a one-phase equivalent.
- As the system is balanced the electric magnitudes of the three phases have the same amplitude and a known phase shift (120°).
- The behaviour of the whole system could be derived from the analysis of the so called **one-phase equivalent** or **phase-neutral equivalent** of the system.



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Analysis of circuits with delta-connected elements

- In systems with a delta-connected load or a delta-connected generator, it is not possible to apply the one-phase equivalent approach directly, since there is no neutral point.
- However it is possible to apply a wye delta transformation to obtain a YY connected system equivalent to the original.
- In Y Δ or $\Delta\Delta$ or Δ Y systems:
 - 1. The system is transformed into a YY equivalent system.
 - 2. The one-phase equivalent approach is applied over the YY equivalent circuit.

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ΔY transformation for delta-connected generators

 Δ connected generators can be redrawn as equivalent Y connected generators:



The two configurations are equivalent if:

$$\underline{\mathbf{U}}_{LY} = \underline{\mathbf{U}}_{L\Delta}$$

ΔY transformation for three delta-connected impedances

We want to find the value of Z_Y that makes the two loads equivalent



The loads are equivalent if for the same applied line voltages the line currents are the same (i.e. $\underline{\mathbf{U}}_{AB\Delta} = \underline{\mathbf{U}}_{ABY} => \underline{\mathbf{I}}_{A\Delta} = \underline{\mathbf{I}}_{AY}$)

ΔY transformation for three delta-connected impedances

The behaviour in the three phases is the same except that there is a phase shift of 120° . We analyse phase A:

$$\underline{\mathbf{I}}_{A\Delta} = \sqrt{3} \cdot \underline{\mathbf{I}}_{PhA} \angle -30^{\circ} = \frac{\underline{\mathbf{U}}_{AB} \cdot \sqrt{3} \angle -30^{\circ}}{Z_{\Delta}}$$

$$\underline{\mathbf{I}}_{AY} = \underline{\mathbf{I}}_{PhA} = \frac{\underline{\mathbf{U}}_{AB}/\sqrt{3}\angle 30^{\circ}}{Z_{Y}}$$

What value of impedance Z_Y verifies the identity $\underline{I}_{A\Delta} = \underline{I}_{AY}$?:

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Instantaneous power

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The instantaneous power of three phase systems is constant*:

$$p(t) = u_a(t) \cdot i_a(t) + u_b(t) \cdot i_b(t) + u_c(t) \cdot i_c(t) = 3 \cdot U \cdot I \cdot \cos \varphi$$

As power is constant the vibrations in the axles of three phase motors and generators are smaller that those in one-phase devices what makes them more stable from the mechanical point of view.

$$u_{a}(t) = \sqrt{2} \cdot U \cdot \cos(\omega t) \qquad i_{a}(t) = \sqrt{2} \cdot I \cdot \cos(\omega t - \varphi)$$
$$u_{b}(t) = \sqrt{2} \cdot U \cdot \cos(\omega t - 120^{\circ}) \qquad i_{b}(t) = \sqrt{2} \cdot I \cdot \cos(\omega t - 120^{\circ} - \varphi)$$
$$u_{c}(t) = \sqrt{2} \cdot U \cdot \cos(\omega t + 120^{\circ}) \qquad i_{c}(t) = \sqrt{2} \cdot I \cdot \cos(\omega t + 120^{\circ} - \varphi)$$

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*Read the demonstration in the long notes

Active and reactive power of a three-phase wye load



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Active and reactive power of a three-phase delta load



 $Z_{\Delta} = |Z_{\Delta}| \angle \theta \qquad \qquad Q = 3 \cdot U_{Ph} \cdot I_{Ph} \cdot \sin \varphi = \sqrt{3} \cdot U_L \cdot I_L \cdot \sin \varphi$

Complex power of three-phase generators



 $\mathcal{S}_{g} = \underline{\mathbf{U}}_{PhA} \cdot \underline{\mathbf{I}}_{PhA}^{*} + \underline{\mathbf{U}}_{PhB} \cdot \underline{\mathbf{I}}_{PhB}^{*} + \underline{\mathbf{U}}_{PhC} \cdot \underline{\mathbf{I}}_{PhC}^{*} = 3 \cdot \underline{\mathbf{U}}_{PhA} \cdot \underline{\mathbf{I}}_{PhA}^{*}$

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Power factor

Power factor: $p.f. = \cos \varphi$



$$\varphi = \varphi_u - \varphi_i = \arctan \frac{Q}{P} = \arctan \frac{X}{R}$$



Reactive power compensation

- Many real-life loads, as electric motors, are highly inductive and often operation of power systems electric systems involves high amounts of reactive power transferred from the generators towards the loads.
- Fluctuating power increases the current flowing through the lines increasing losses and giving rise to voltage drops.
- Electric companies penalize the costumers that consume power with poor power factor.



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Reactive power compensation

Banks of capacitors are connected in parallel with the loads, to compensate part of the reactive power absorbed by them.



Reactive power compensation

- Capacitors do not absorb or deliver any active power, so the active power of the system remains unchanged. $Q_C < 0$
- The relation between the active an reactive power changes and the angle φ' becomes smaller.
- The power factor becomes closer to 1

Initial system

System with capacitors





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 $Q' = Q + Q_C$

Reactive power of a capacitor

A capacitor of capacitance C, a voltage drop $\underline{\mathbf{U}} = U \angle \varphi_u$ and current flow $\underline{\mathbf{I}} = I \angle \varphi_i$



$$Q_C = X_C \cdot I^2 = \frac{U^2}{X_C} = -\omega \cdot C \cdot U^2$$

 $P_C = 0$

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Reactive power of a bank of capacitors in wye



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Reactive power of a bank of capacitors in delta



Capacitance required to get a target power factor

We want to compensate the reactive power of a system working with power factor $\cos \varphi$ so that the power factor becomes $\cos \varphi'$



$$C_{\Delta} = \frac{P \cdot (\tan \varphi - \tan \varphi')}{3 \cdot \omega \cdot U_L^2} \qquad C_Y = \frac{P \cdot (\tan \varphi - \tan \varphi')}{\omega \cdot U_L^2}$$

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Measure of power: working principle of wattmeters

A wattmeter is a measuring device that provides information on the power absorbed by electric dipoles.

Wattmeters incorporate two measuring circuits: the current coil and the voltage coil.

A sign * marks the terminals of the current and voltage coils of the same polarity



$$W = I_A \cdot U_{AB} \cdot \cos(\widehat{\mathbf{U}_{AB}\mathbf{I}_A})$$

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Measure of the active power in systems with accessible neutral point



$$W = I_A \cdot U_{AN} \cdot \cos(\widehat{\mathbf{U}_{AN}\mathbf{I}_A}) = U_{Ph} \cdot I_{Ph} \cdot \cos\varphi = \frac{P}{3}$$

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Measure of the reactive power of a three-phase system with one wattmeter



$$W = U_{BC} \cdot I_A \cdot \cos(\widehat{\mathbf{U}_{BC}\mathbf{I}_A})$$

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Measure of the reactive power of a three-phase system with one wattmeter



$$W = U_L \cdot I_L \cdot \cos(\widehat{\underline{\mathbf{U}}_{BC}}\underline{\mathbf{I}}_A) = U_L \cdot I_A \cdot \cos(90 - \varphi) = U_L \cdot I_A \cdot \sin(\varphi) = \frac{Q}{\sqrt{3}}$$

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The two wattmeters method



$$W_1 = U_{AC} \cdot I_A \cdot \cos(\widehat{\mathbf{U}}_{AC}\mathbf{I}_A)$$

$$W_2 = U_{BC} \cdot I_B \cdot \cos(\widehat{\mathbf{U}_{BC}\mathbf{I}_B})$$

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The two wattmeters method

$$W_1 = U_L \cdot I_L \cdot \cos(\widehat{\mathbf{U}_{AC}\mathbf{I}_A}) = U_L \cdot I_L \cdot \cos(30 - \varphi) = U_L \cdot I_L \cdot (\frac{\sqrt{3}}{2}\cos\varphi + \frac{1}{2}\cdot\sin\varphi)$$

$$W_2 = U_L \cdot I_L \cdot \cos(\widehat{\mathbf{U}_{BC}\mathbf{I}_B}) = U_L \cdot I_L \cdot \cos(30 + \varphi) = U_L \cdot I_L \cdot (\frac{\sqrt{3}}{2}\cos\varphi - \frac{1}{2}\cdot\sin\varphi)$$

The active and reactive power of the three phase system can be obtained as the sum and the difference of the measures of the two wattmeters.

$$W_1 + W_2 = \sqrt{3} \cdot U_L \cdot I_L \cdot \cos \varphi = P$$

$$W_1 - W_2 = \cdot U_L \cdot I_L \cdot \sin \varphi = \frac{Q}{\sqrt{3}}$$

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