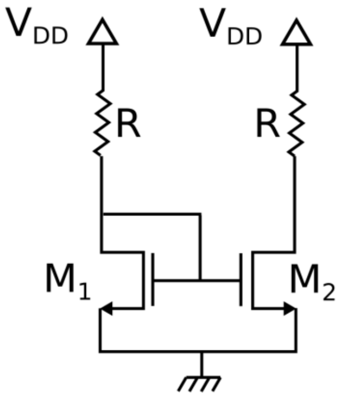
Unit 3: EXERCISES

Ex1. For the following circuit, compute the current of the transistors (L = 1µm).



Data: V_{DD} = 5V, R =100 kΩ, $\mu_n C_{ox}$ = 120 µA/V², $(W/L)_1=(W/L)_2=5$, V_{th} = 0.8V.

Neglecting channel modulation and assuming saturation for both M_1 and M_2:

$$I_{M1} = \frac{V_{DD} - V_{GS,M1}}{R}$$

$$I_{M1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})^2$$

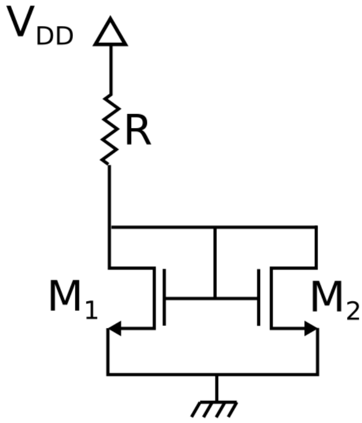
➔ $V_{GS,M1} = 1.16V, I_{M1} = 39\ \mu A$

M1 and M2 have the same size ratio, so:

$$I_{M1} = I_{M2} = 39\ \mu A$$

In the file ex1_current_mirror.asc you can see a simulation of this example, showing similar results than the theoretical ones.

Ex2. Repeat Ex1 for the following circuit:



Data: V_{DD} = 5V, R =100 kΩ, $\mu_n C_{ox}$ = 120 µA/V^2, $(W/L)_1=(W/L)_2=5$, V_{th} = 0.8V.

Neglecting channel modulation and assuming saturation for both M_1 and M_2:

$$I_{M1} + I_{M2} = \frac{V_{DD} - V_{GS,M1}}{R}$$

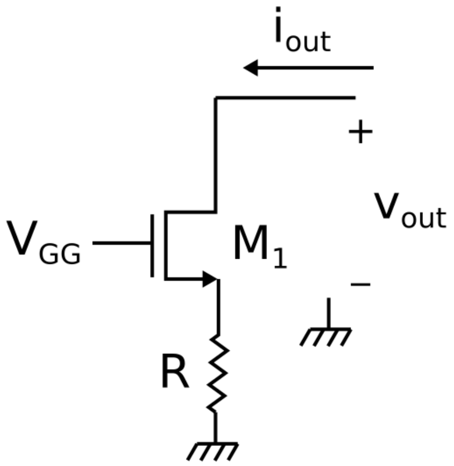
$$I_{M1} = I_{M2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})^2$$

$$\rightarrow V_{GS,M1} = 1.16V, I_{M1} = I_{M2} = 19.5\ \mu A$$

Is it possible to replace M_1 and M_2 with one single device?

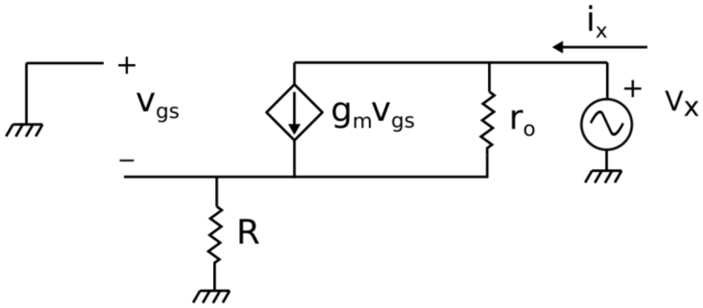
Yes, we can use one single device with twice the size ratio → (W/L) = 10.

Ex3. Compute the output resistance of the following current source:



Data: R =50 kΩ, $\mu_n C_{ox}$ = 110 μA/V², $(W/L)_1$=2, $i_{out,DC}$ = 10 μA, λ=0.04 V⁻¹

Using the small-signal model of M_1:



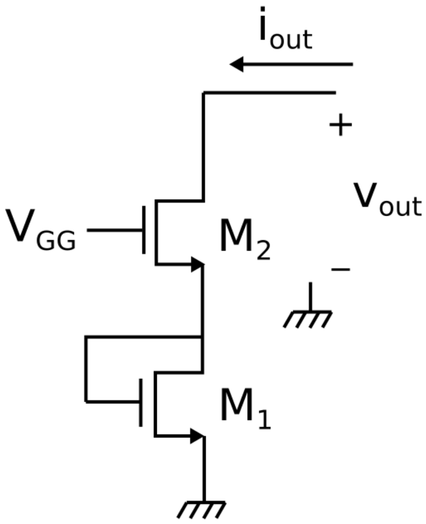
$$r_{out} = \frac{v_x}{i_x} = R + r_o + g_m r_o R$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = 66.33\frac{\mu A}{V}$$

$$r_o = \frac{1}{\lambda I_D} = 2.5\ M\Omega$$

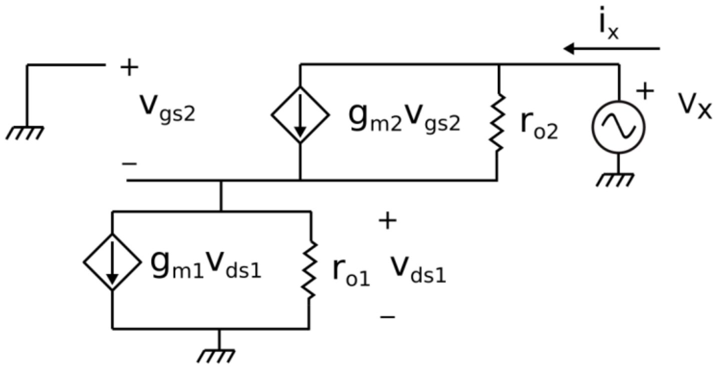
$$r_{out} = 10.84\ M\Omega$$

Ex4. Making use of the following circuit, compute $(W/L)_{M1}$ to have the same output resistance as in Ex3.



Data: $\mu_n C_{ox}$ = 110 μA/V², $(W/L)_2$=2, $i_{out,DC}$ = 10 μA, λ=0.04 V^{-1}

Using the small-signal model:



$$r_{out} = \frac{v_x}{i_x} = \frac{r_{o1} r_{o2}(g_{m1} + g_{m2}) + r_{o1} + r_{o2}}{1 + g_{m1} r_{o1}}$$

$$g_{m2} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 66.33 \frac{\mu A}{V}$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = 2.5\ M\Omega$$

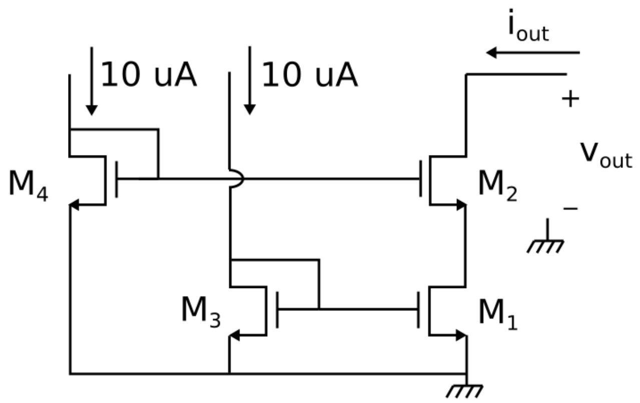
$$r_{out} = 11\ M\Omega$$

➔ $g_{m1} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 19.22 \frac{\mu A}{V}$

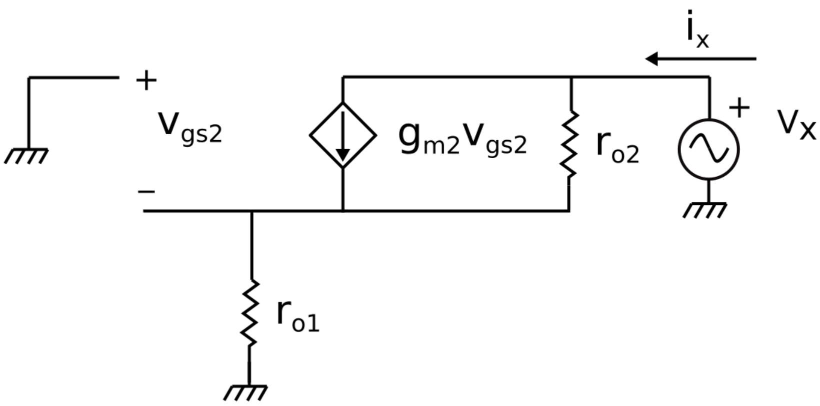
$$\left(\frac{W}{L}\right)_1 \approx 1/6$$

For instance, we can choose: W = 1 μm, L = 6 μm.

Ex5. Compute the output resistance of the following current source.



Data: $\mu_n C_{ox}$ = 110 $\mu A/V^2$, $(W/L)_1=(W/L)_2=(W/L)_3=4$, $(W/L)_1=1$, $\lambda=0.04\ V^{-1}$



Using the small-signal model:

$$r_{out} = \frac{v_x}{i_x} = r_{o1} + r_{o2} + g_{m1} r_{o1} r_{o2}$$

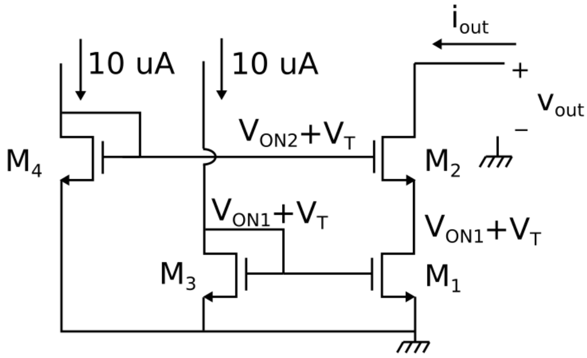
$$i_{out} = 10\ \mu A$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 93.8 \frac{\mu A}{V}$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = 2.5\ M\Omega$$

$$r_{out} = 591\ M\Omega$$

What is the minimum V_{out} we may have?



Where V_{ON} is the overdrive voltage → $V_{ON} = \sqrt{\frac{2I_D}{\mu nCox(W/L)}}$

$$V_{D2} \geq V_{G2} - V_T$$

$$V_{out,min} = V_{ON2}$$

With $I_D = 10\ \mu A$

➔ $V_{out,min} = 0.426\ V$

Exercises adapted from:

- Allen, P. E., & Holberg, D. R. (2011). CMOS analog circuit design. New York: Oxford University Press, USA.
- R Jacob Baker 2010 CMOS Circuit Design, Layout, and Simulation 3 rd ed Wiley IEEE Press