

# Symmetric encryption: Block ciphers

## CRYPTOGRAPHY AND COMPUTER SECURITY

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COSEC



# OUTLINE

- 5. Symmetric encryption: Block ciphers
  - Modern encryption
  - Block ciphers
    - Introduction
    - Feistel scheme
    - Operation modes
    - Block ciphers: advantages and disadvantages
    - DES
    - AES

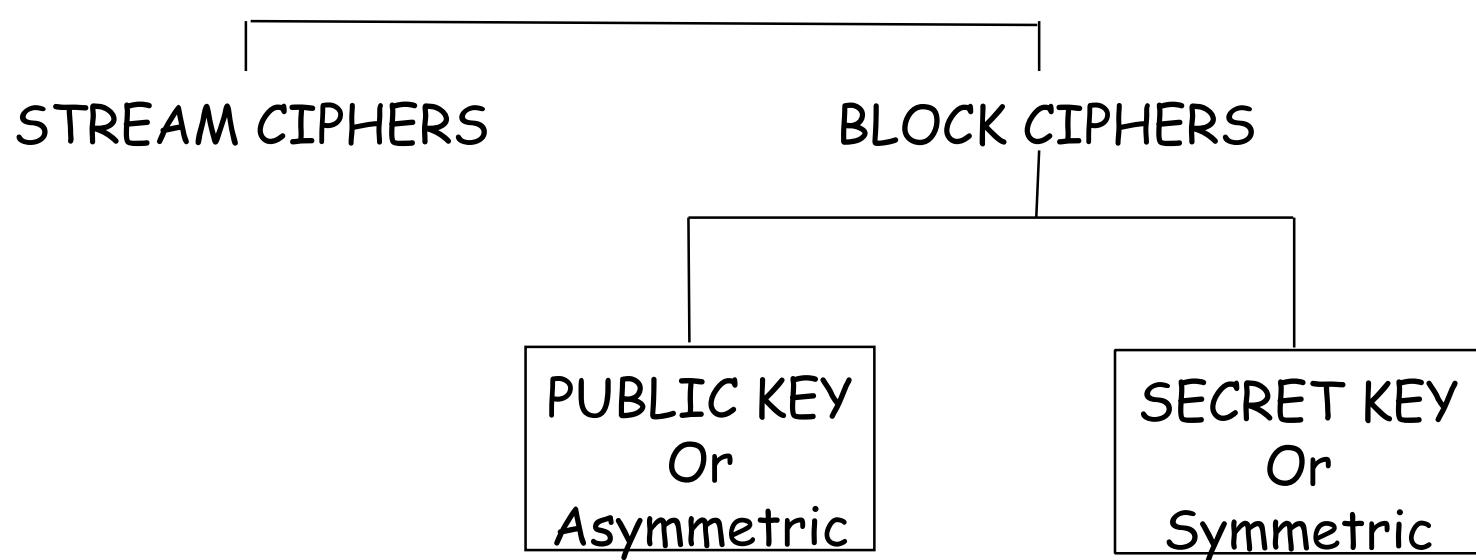
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# Modern encryption

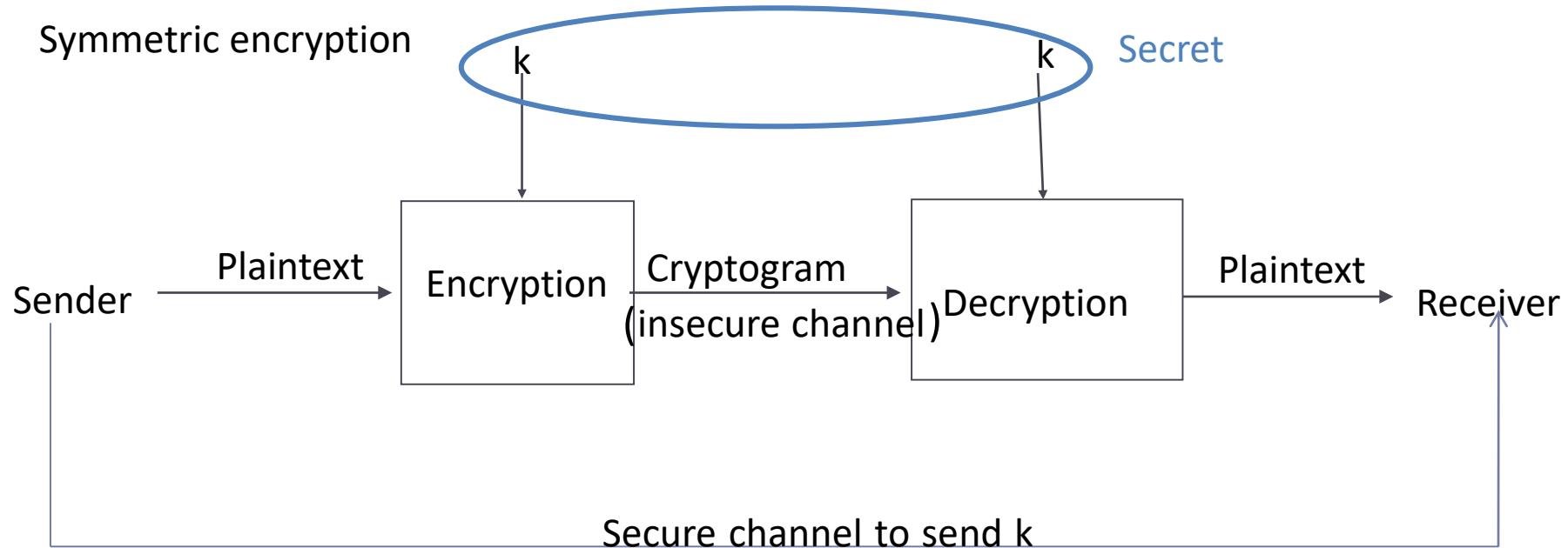
- Classification
  - Type of operations
    - In general, substitutions and transpositions
  - According to the key used:
    - Symmetric (Secret key)
    - Asymmetric (Public key)
  - According to the number of symbols encrypted at a time
    - Stream (1 symbol or a few)
    - Block (a set of symbols at a time)

# Modern encryption



# Modern encryption

Symmetric encryption

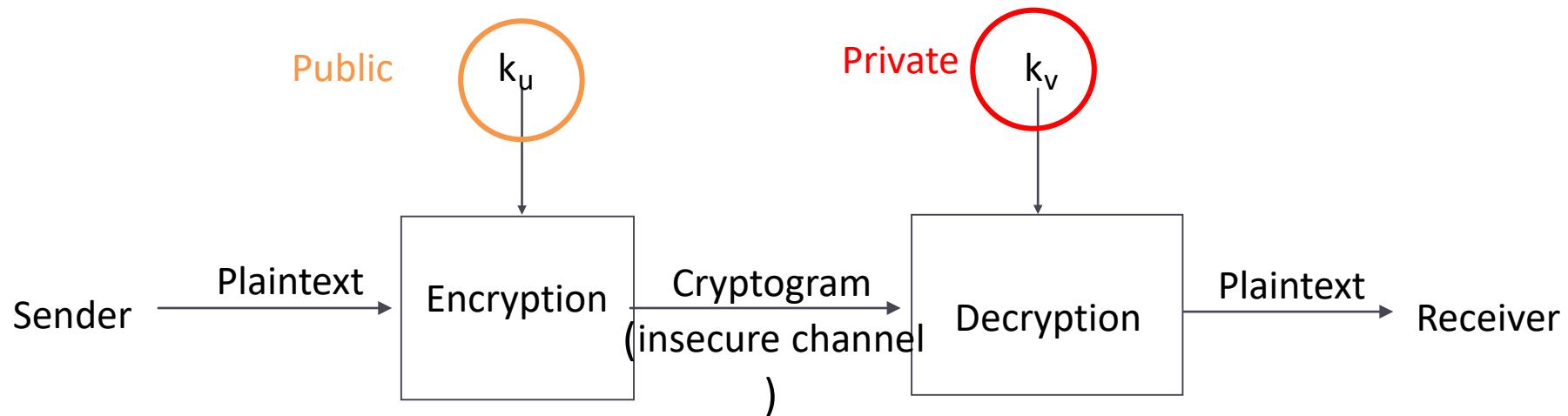


$$C = E(k, M) = E_k(M)$$

$$M = D(k, C) = D_k(C)$$

# Modern encryption

## Asymmetric encryption



$$C = E(k_u, M) = E_{ku}(M)$$

$$M = D(k_v, C) = D_{kv}(C)$$

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# Block ciphers. Introduction

- $M$  is divided in blocks of equal length:  
 $M_1, M_2, \dots M_n$
- Each block is encrypted with the same key  
 $C = E_k(M) = E_k(M_1) E_k(M_2) \dots E_k(M_n)$
- Typical block sizes 64, 128 or 256 bits
- Reversible mapping between  $M$  and  $C$  blocks

# Block ciphers. Introduction

- Substitution of very long “characters”
  - 64 bits or more
- Ideal block cipher
  - $n$ : block size. E.g.: 64
  - Substitution tables (mapping) of  $2^n$  bits
  - $2^n!$  possible keys (matches  $C_i$ )
  - Not practical
    - Substitution table is the key, length =  $n \cdot 2^n$  bits
    - For  $n = 64 \rightarrow$  key length  $10^{21}$  bits aprox.

# Block ciphers. Introduction

Algorithm	Block size (bits)	Key size (bits)	Rounds
Lucifer	128	128	16
DES	64	56	16
Twofish	128	variable	variable
RC2	64	variable	18
RC5	variable	variable	variable
SAFER	64	64	8
IDEA	64	128	8
Skipjack	64	80	32
RIJNDAEL	128	128 or more	flexible

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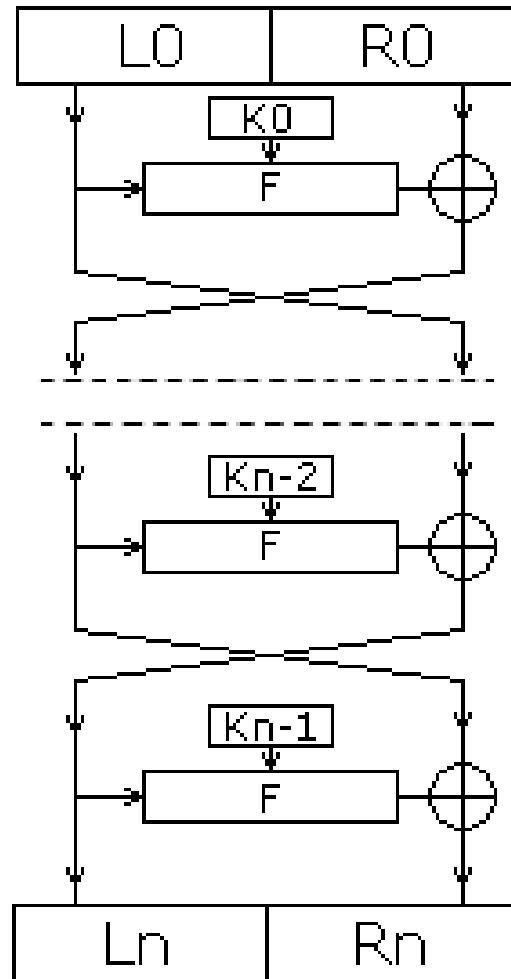
# Block ciphers. Feistel scheme

- Block cipher (Feistel 1975)
  - Confine to a subset of  $2^n!$  possible keys
    - n: block text size
    - k: key size
    - $2^k$  possible keys
  - Product cipher
    - Substitution (S-box)
    - Permutation (P-box)
  - Practical application of Shannon's proposal (1949)
    - High *diffusion*
    - High *confusion*

# Block ciphers. Feistel scheme

- Methods to thwart cryptanalysis
  - Diffusion
    - statistical structure of M is dissipated in C
    - each C bit is affected by many M bits
    - achieved performing some permutation on  $M_i$  followed by a function to that permutation
  - Confusion
    - seeks to make  $C - k$  statistical relationship as complex as poss.
    - achieved by the use of a complex substitution algorithm

# Block ciphers. Feistel scheme



# Block ciphers. Feistel scheme

- Divide the block into two halves  $L_0$  and  $R_0$
- Substitute the left half
  - Apply a *round function*  $F$  (non linear) to the right half of the data and then XOR the output and the left half
    - $F$  is a function of the right half and the round subkey  $k_i$
- Permute the two halves
- Repeat it  $n$  **rounds**

# Block ciphers. Feistel scheme

- Same circuit to encrypt and decrypt
  - Just use the subkeys in reverse order
  - A final permutation is needed (fig. slide 15)
    - $L_{n+1} = R_n$
    - $R_{n+1} = L_n$
- In practice the design problem is reduced to:
  - Develop a good subkey generation algorithm
  - Develop a good round function F
- Many b.c. follows Feistel scheme but not all

# Block ciphers. Feistel scheme

- Block size
  - Larger size, greater security, lower speed
  - 64 or more
- Key size
  - Larger size, greater security, lower speed
  - 128 or more
- Number of rounds
  - Higher number, greater security, lower speed
  - Typical value 16
- Subkey generation algorithm and round function F
  - Greater complexity, greater resistance to cryptoanalysis



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# Block ciphers. Operation modes

- Technique for enhancing the effect of a cryptographic algorithm or adapting the algorithm for an application
- Intended for use with any symmetric block cipher
- Five modes defined by NIST (SP 800-38A)

Electronic Code Book ECB

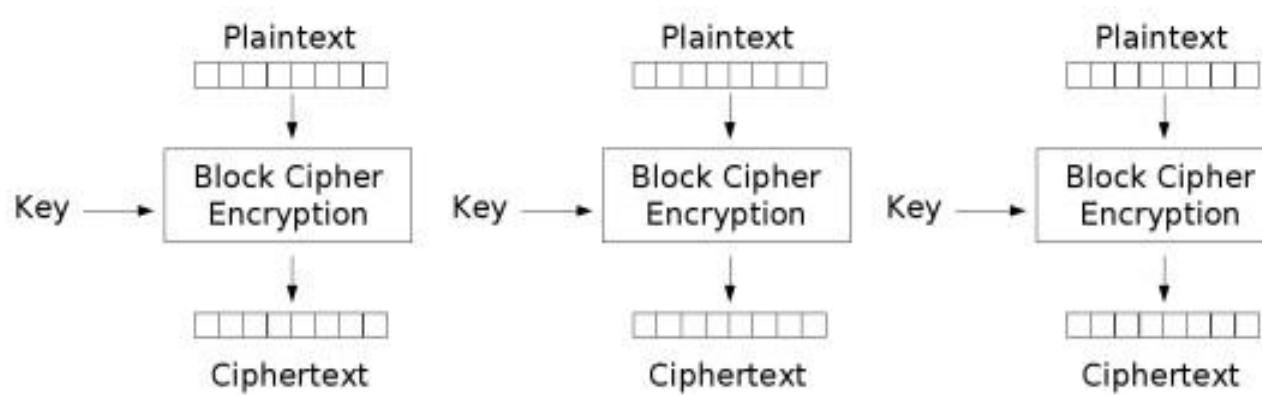
Cipher Block Chaining CBC

Cipher Feedback CFB

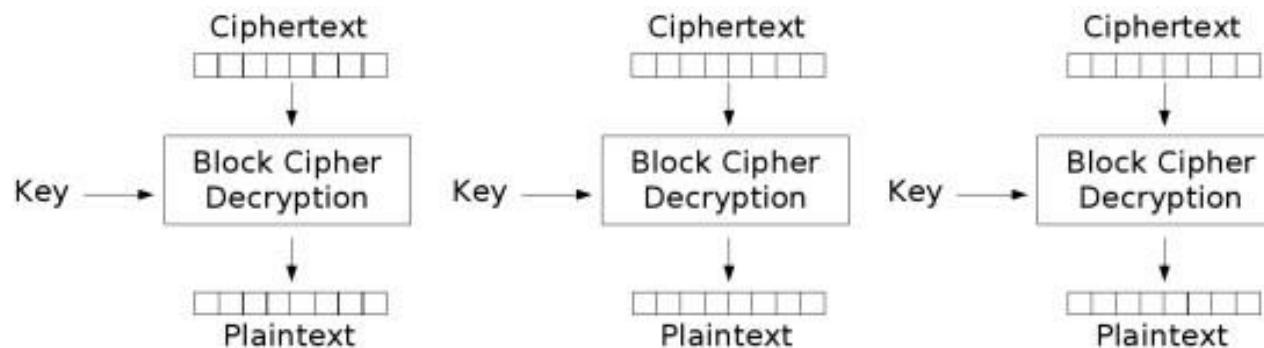
Output Feedback OFB

Counter Mode CTR (recommended)

# Block ciphers. Electronic Code Book mode (ECB)



Electronic Codebook (ECB) mode encryption

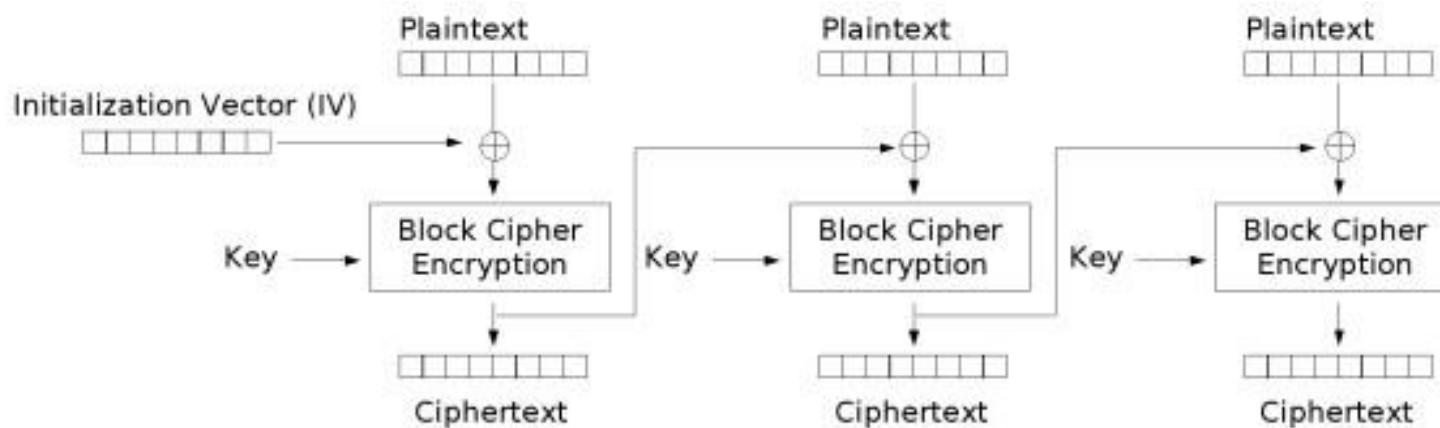


Electronic Codebook (ECB) mode decryption

# Block ciphers. Electronic Code Book mode (ECB)

- Advantages:
  - Block encryption and decryption can be executed in parallel } Ideal for a short amount of data
  - e.g. symmetric key
  - Bit errors in transmission do not propagate
- Disadvantages:
  - Repeated plaintext blocks produce repeated ciphertext blocks
  - It is possible to modify the order of the blocks or eliminate them
  - ***Padding*** of the last block is necessary
  - E.g.: add zero bytes and a last byte reporting #padding\_bytes

# Block ciphers. Electronic Code Book mode (ECB)

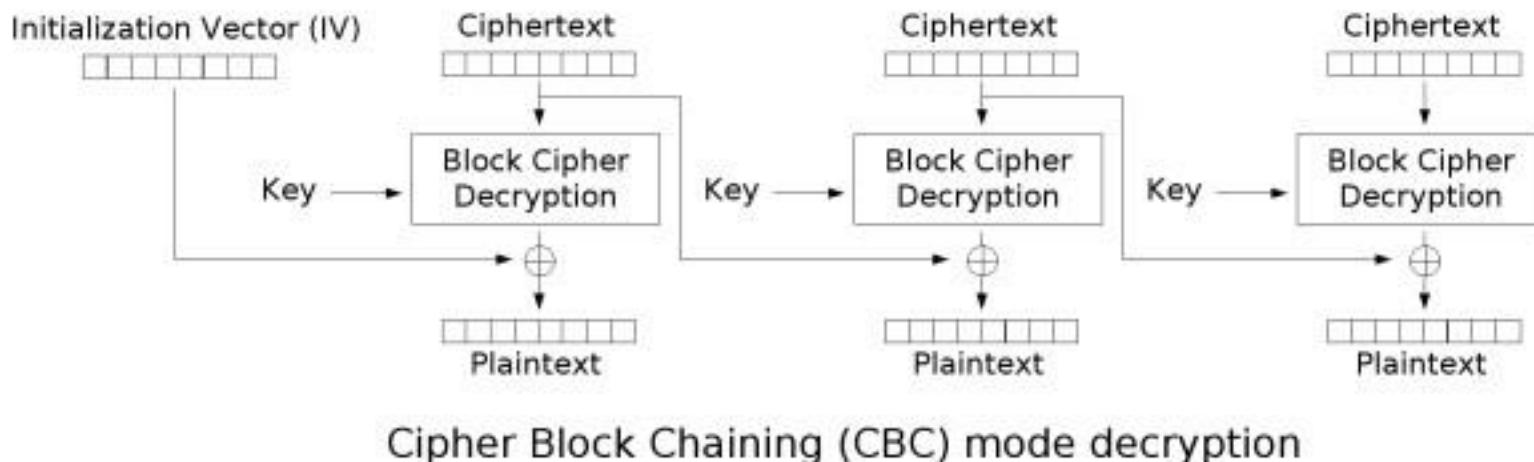


Cipher Block Chaining (CBC) mode encryption

$$C_i = E_K(P_i \oplus C_{i-1}), C_0 = IV$$

- IV confidential to parties (integrity reasons)

# Block ciphers. Cipher Block Chaining mode (CBC)



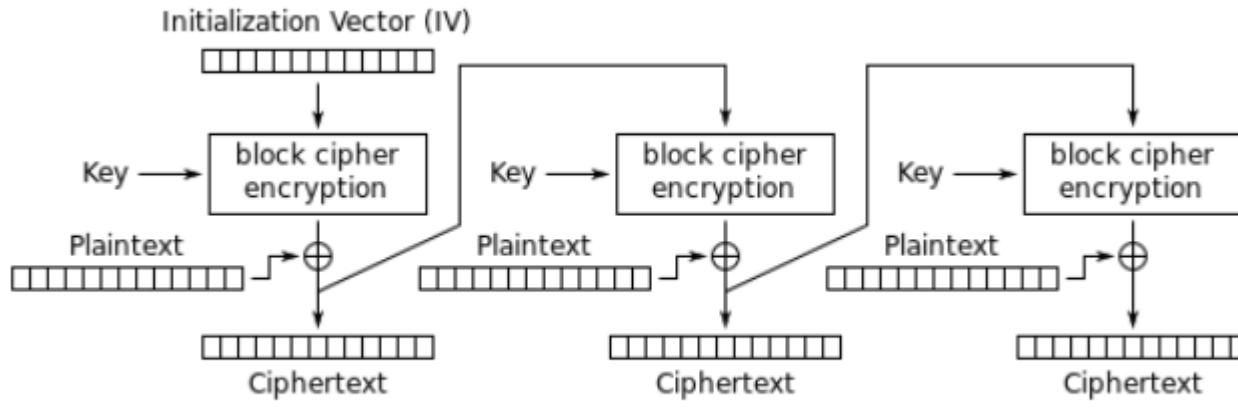
$$P_i = D_K(C_i) \oplus C_{i-1}, C_0 = IV$$

- A bit error in transmission affects two Mi
- ***Padding*** needed

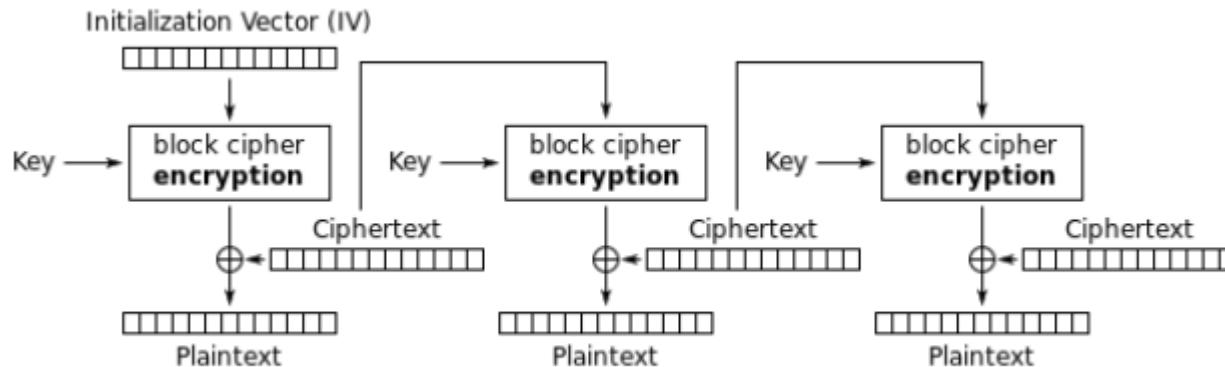
# Block ciphers. Cipher Feedback mode (CFB)

- Uses a shift register
- Plaintext is divided in *segments* (smaller than blocks)
- A bit error in transmission affects two Mi
- Converts a block cipher into a stream cipher
  - But keystream depends on the plaintext

# Block ciphers. Cipher Feedback mode (CFB)

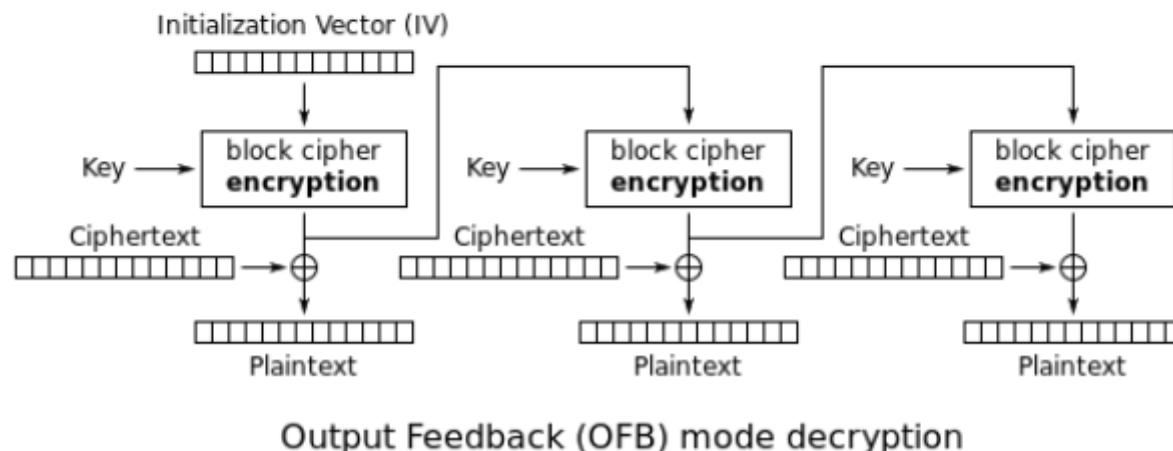
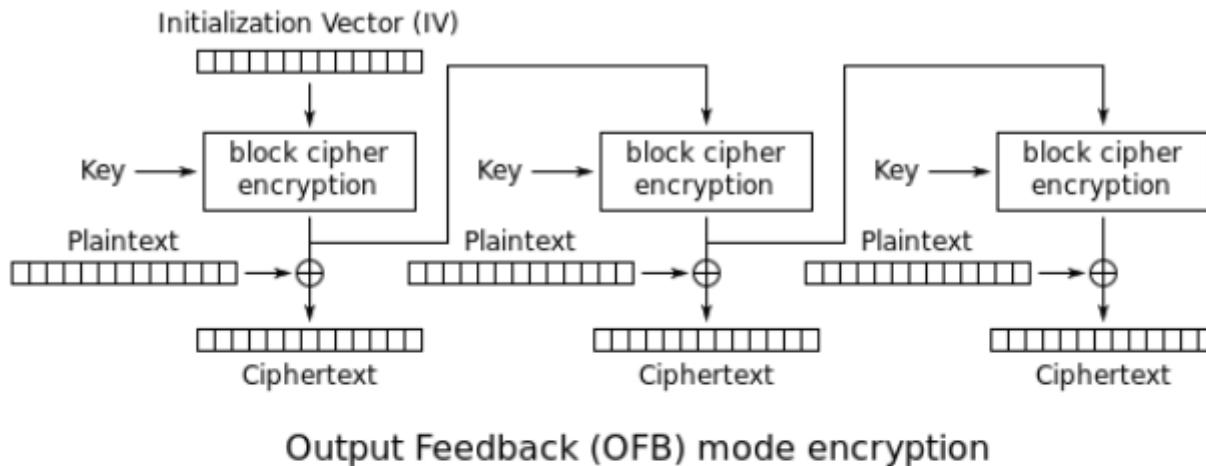


Cipher Feedback (CFB) mode encryption



Cipher Feedback (CFB) mode decryption

# Block ciphers. Output Feedback mode (OFB)



# Block ciphers. Output Feedback mode (OFB)

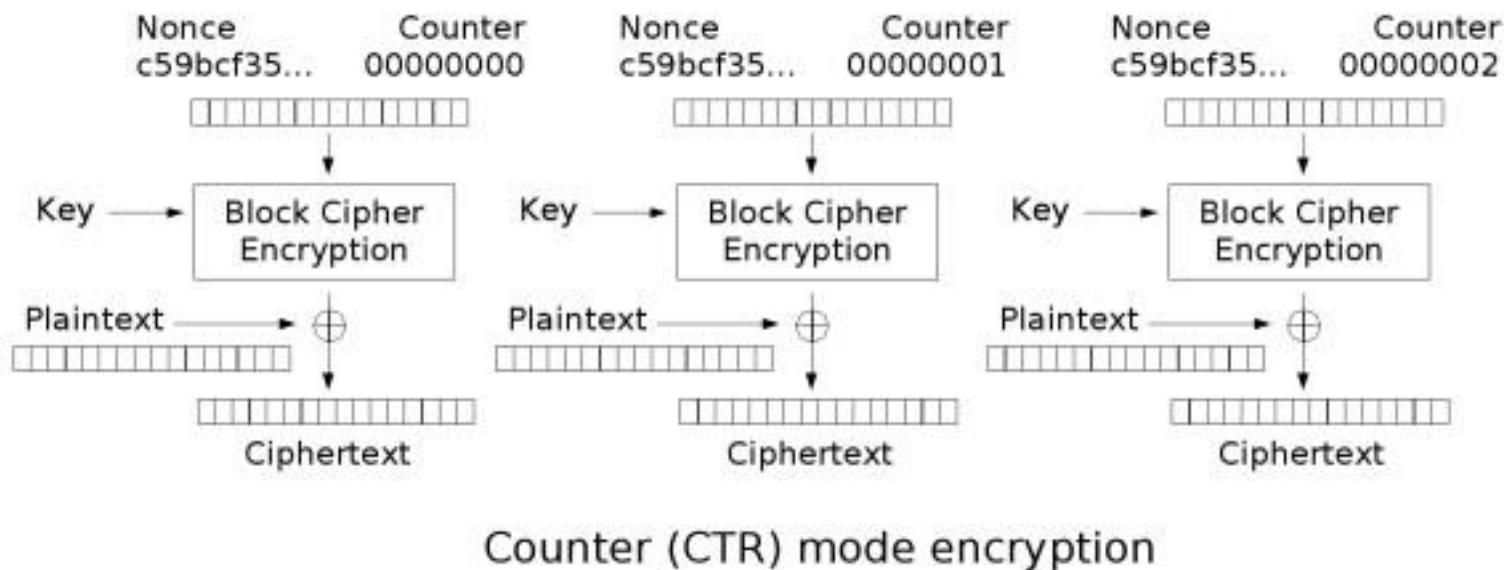
- IV must be a *nonce*
- Bit error in transmission do not propagate
- Just affects a single bit of an  $M_i$
- Does not need padding
- Remaining bits of the last output block are discarded
- Converts a block cipher into a stream cipher
  - Keystream does not depend on the plaintext
  - Works over blocks not over segments

# Block ciphers. Counter mode (CTR)

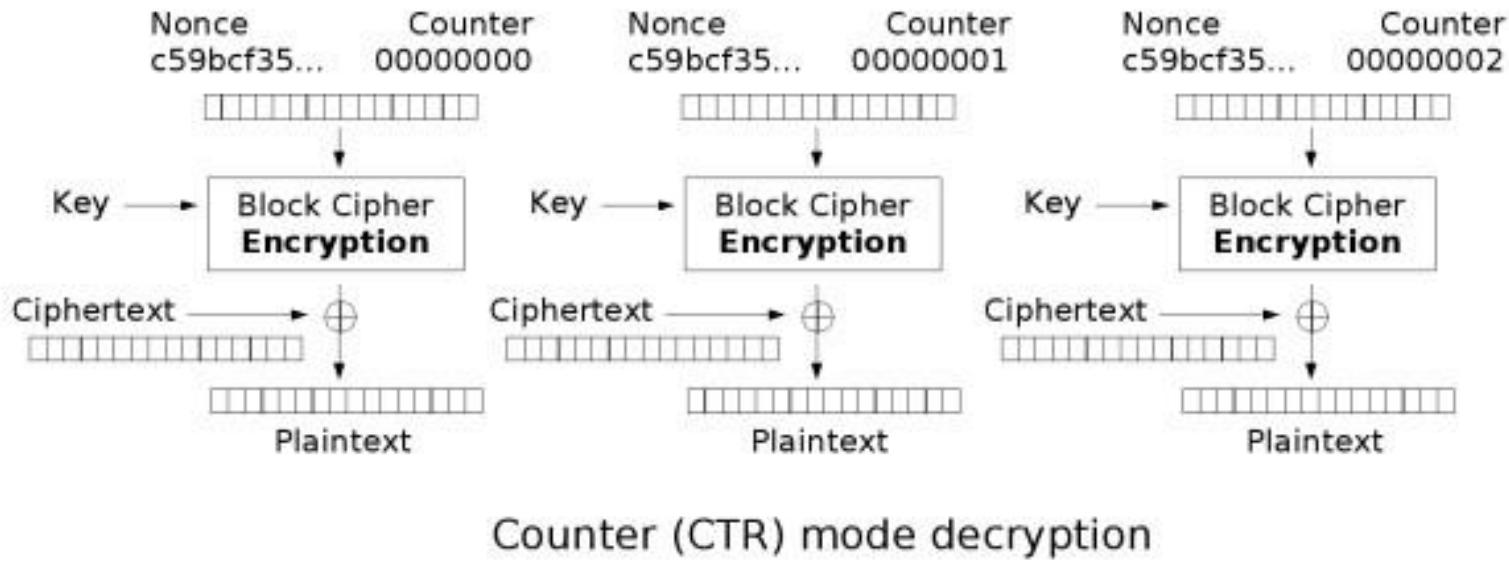
- Uses a counter of the size of a block ( $n$ )
- Incremented by  $1 \bmod 2^n$  across consecutive blocks
- Remaining bits of the last output block are discarded
- Converts a block cipher into a stream cipher
  - Keystream does not depend on the plaintext
  - Works over blocks not over segments

# Block ciphers. Counter mode (CTR)

- Simplicity and random access



# Block ciphers. Counter mode (CTR)



- A bit error in transmission does not propagate
- It only affects a single bit of a block

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# Block ciphers. Advantages and disadvantages

- Use: confidentiality
- Advantages:
  - High diffusion and confusion
  - Simple implementation
  - Symmetry
    - Similar encryption and decryption processes
    - Same circuits to encrypt and decrypt (not always, e.g. AES)
  - Efficiency
    - Fast process

# Block ciphers. Advantages and disadvantages

- Disadvantages:
  - Secure channel is required (key distribution)
  - Management of a high number of keys
  - Effectiveness
    - Slower than stream ciphers, the whole block should be read
    - If  $M$  length is not a multiple of the block size,  $C$  length is bigger
  - Security and robustness
    - Error propagation
    - Vulnerable to attacks if blocks are repeated
  - Padding gives clues to cryptanalysts



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# Block ciphers. Data Encryption Standard (DES)

- 1971 LUCIFER: IBM research project finishes (Feistel)
  - Key size: 128 bits
- 1974: NBS (now NIST) request for proposals for a national cipher standard
- 1976: A modified version of LUCIFER wins
  - Key size reduced to 56 in order to fit on a single chip
  - NSA changed de S-boxes
- 1977: DES standard for commercial, bank and unclassified communications
- 1983, 1988, 1993: NIST reaffirmed DES as a standard
  - Criticism
  - Key length
  - Obscure design
    - Suspicions of the National Security Agency (NSA)

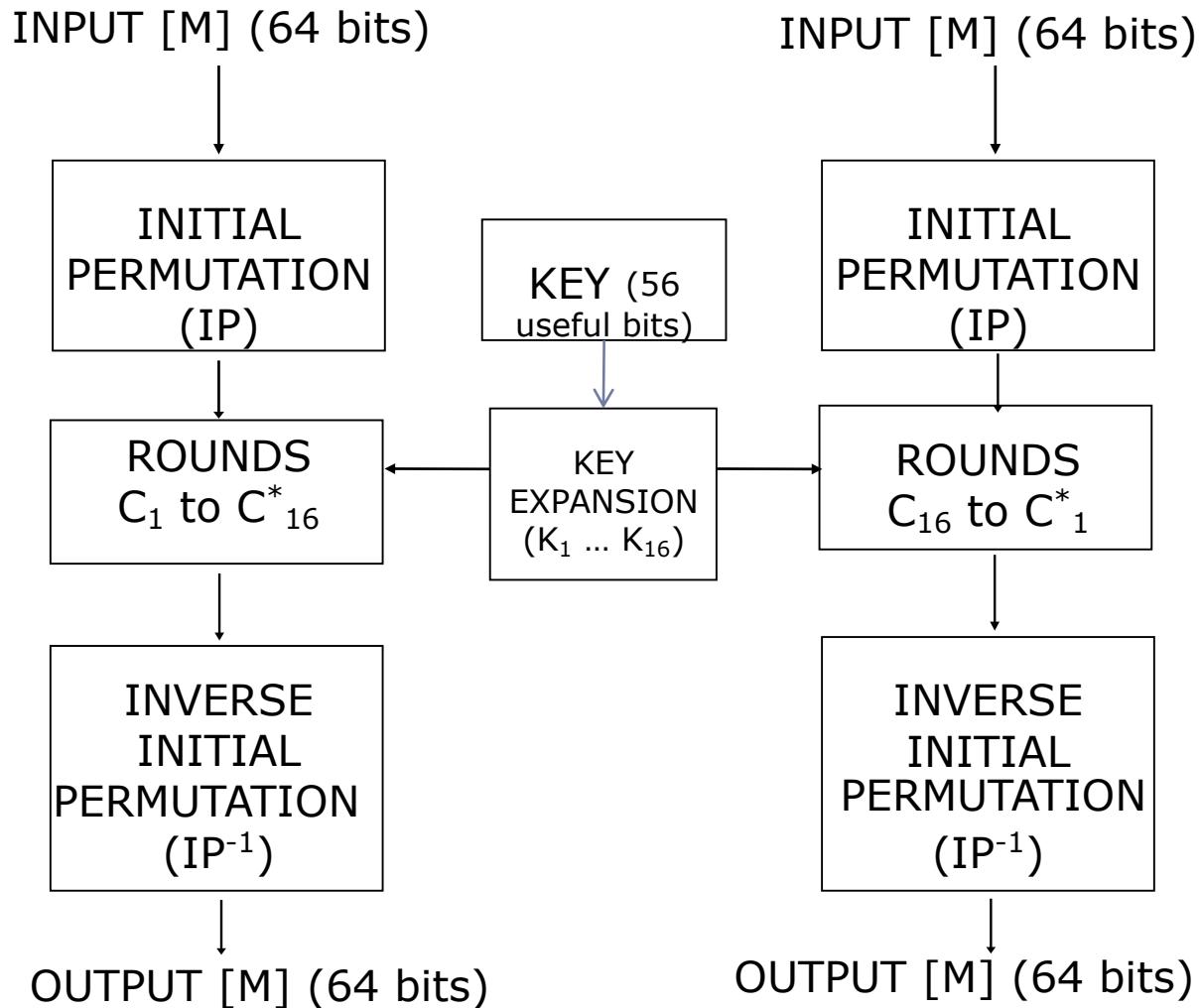


# Block ciphers. Data Encryption Standard (DES)

- 1990: Differential cryptoanalysis (Biham and Shamir)
  - $2^{47}$  chosen ciphertexts needed. Effort on  $2^{47}$  encryptions
  - Lucifer was vulnerable but DES is not
- 1993: Linear Cryptanalysis (Matsui)
  - $2^{43}$  known plaintexts needed
- 1998: DES Cracker de la Electronic Frontier Foundation
  - 56 hours
  - Using 1536 dedicated chips
  - \$250K, less than a year to build it
- 1999: DES Cracker version 2
  - 22 hours
  - Combines 100K PCs
- 1999: Triple DES as new standard
  - DES just for legacy systems
- **2001: new contest and new standard -> AES (Advanced Encryption Standard)**

# DES: Encryption and decryption scheme

- Key: 64 bits
  - (8 parity bits)
- Block size: 64 bits
- Rounds: 16
  - Last one needs one additional permutation (\*)
- Internal keys:
  - 16 48-bits keys
- Mathematical basis:
  - substitutions
    - lineal
    - non lineal
  - permutations



# OUTLINE

- (...)
  - Data Encryption Standard (DES)
    - Encryption
    - Key expansion
    - Decryption
    - Triple DES
    - Security
  - AES
  - (...)

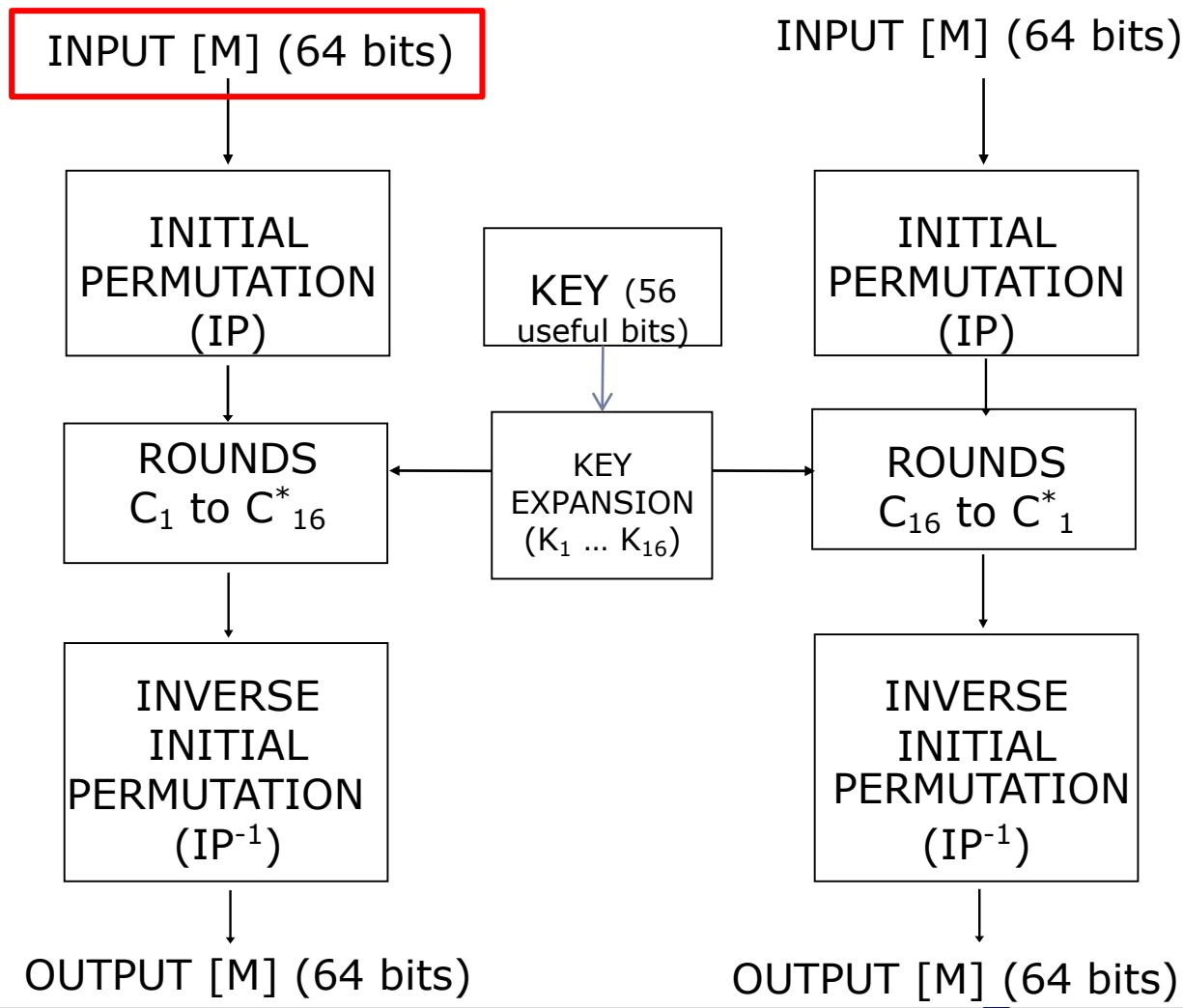
# DES: Algorithm description

## Cleartext

1. Select block M to be encrypted
2. Place the 64 bits of the cleartext as follows

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

# DES: “Situational map”



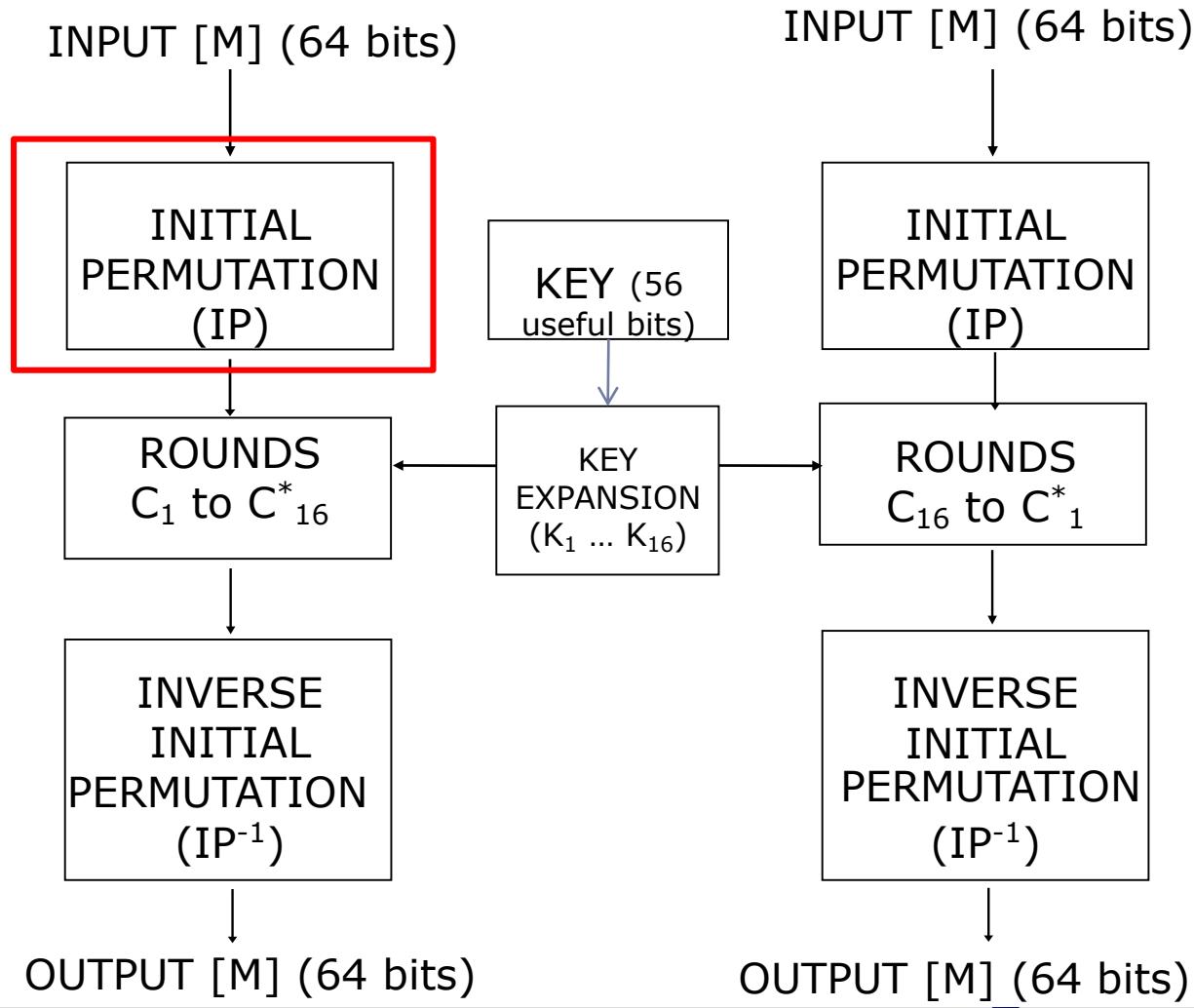
# DES: Algorithm description

## From cleartext to initial permutation

### 3. Initial Permutation, IP

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

# DES: “Situational map”



# DES: Algorithm description

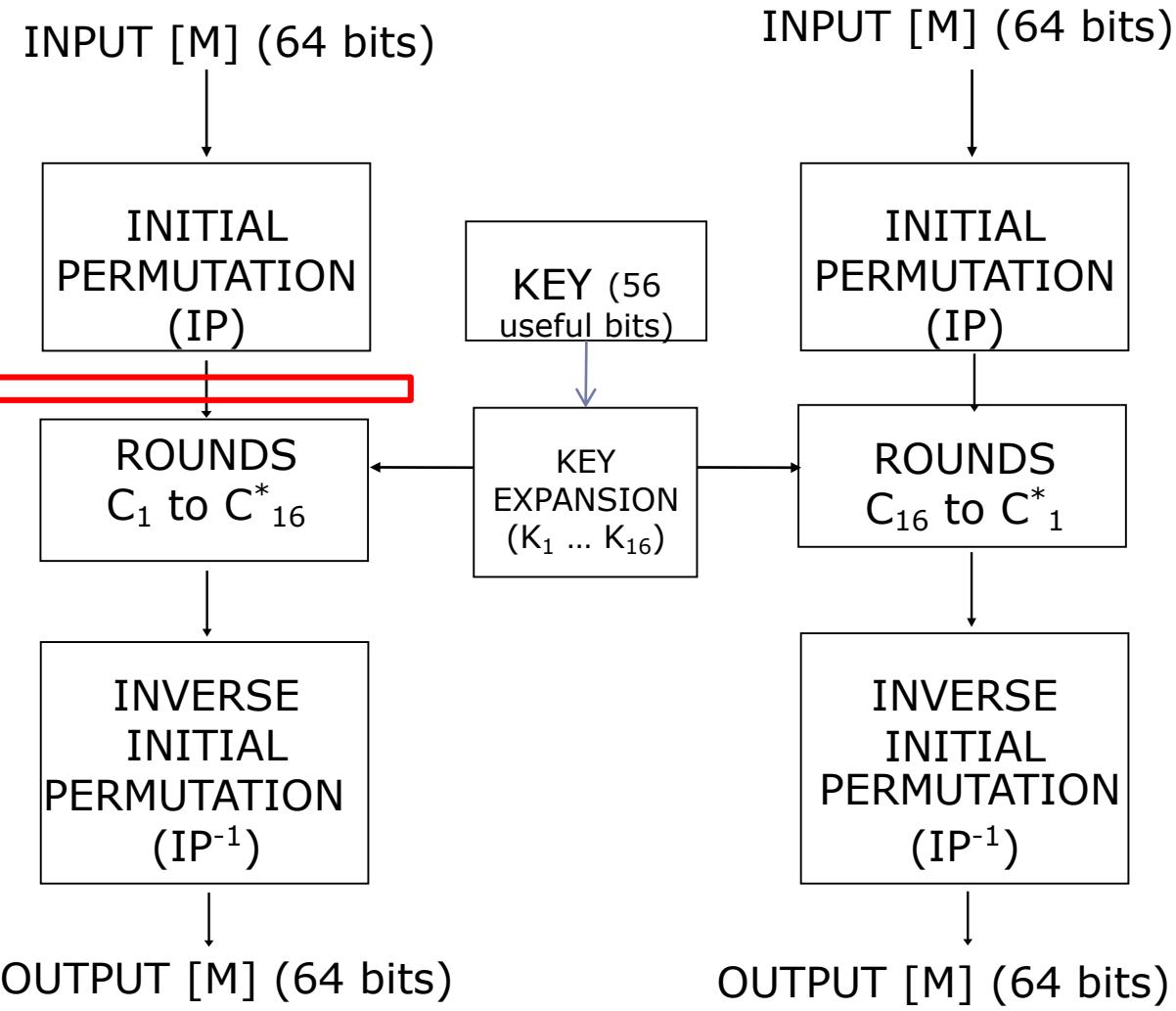
## Operation before the initial round

4. Computation of left and right sub halves, L<sub>0</sub> and R<sub>0</sub>, of 32 bits each

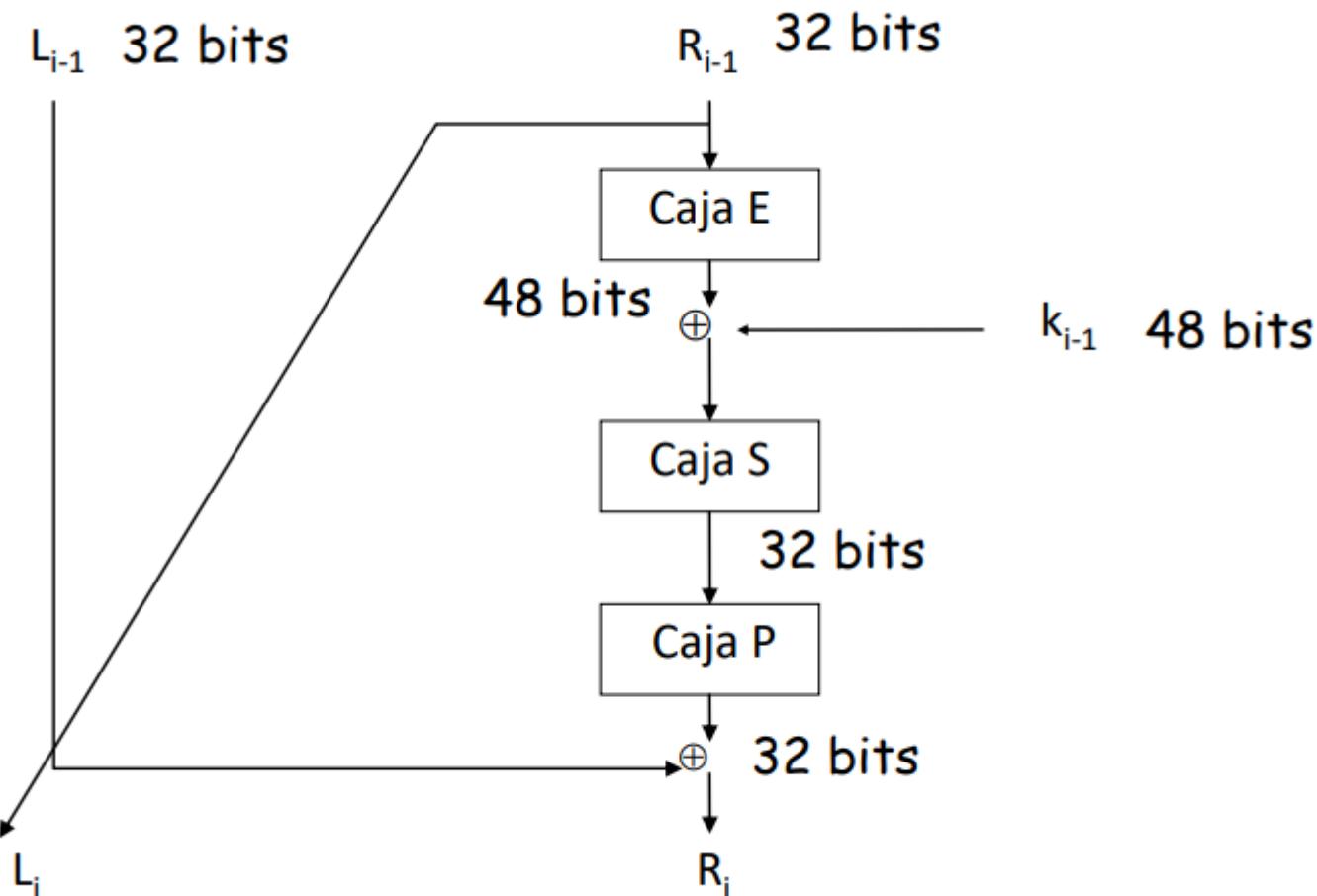
58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

$$L_0 = 58\ 50\ 42\ 34\ 26\ 18\ 10\ 02\ 60\ 52\ 44\\ 36\ 28\ 20\ 12\ 04\ 62\ 54\ 46\ 38\ 30\ 22\ 14\\ 06\ 64\ 56\ 48\ 40\ 32\ 24\ 16\ 08$$
$$R_0 = 57\ 49\ 41\ 33\ 25\ 17\ 09\ 01\ 59\ 51\\ 43\ 35\ 27\ 19\ 11\ 03\ 61\ 53\ 45\ 37\ 29\ 21\\ 13\ 05\ 63\ 55\ 47\ 39\ 31\ 23\ 15\ 07$$

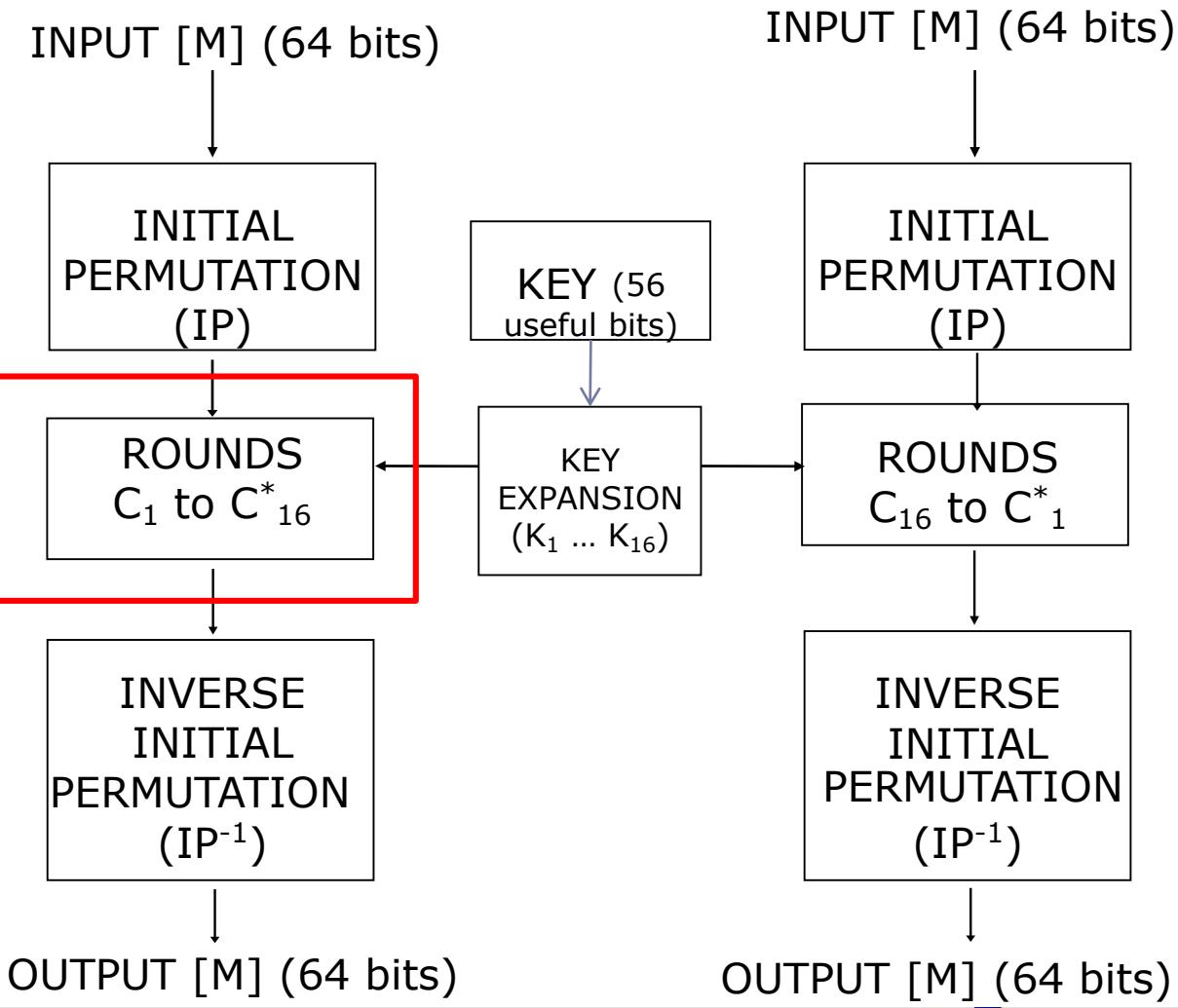
# DES: “Situational map”



# DES: Round scheme



# DES: “Situational map”

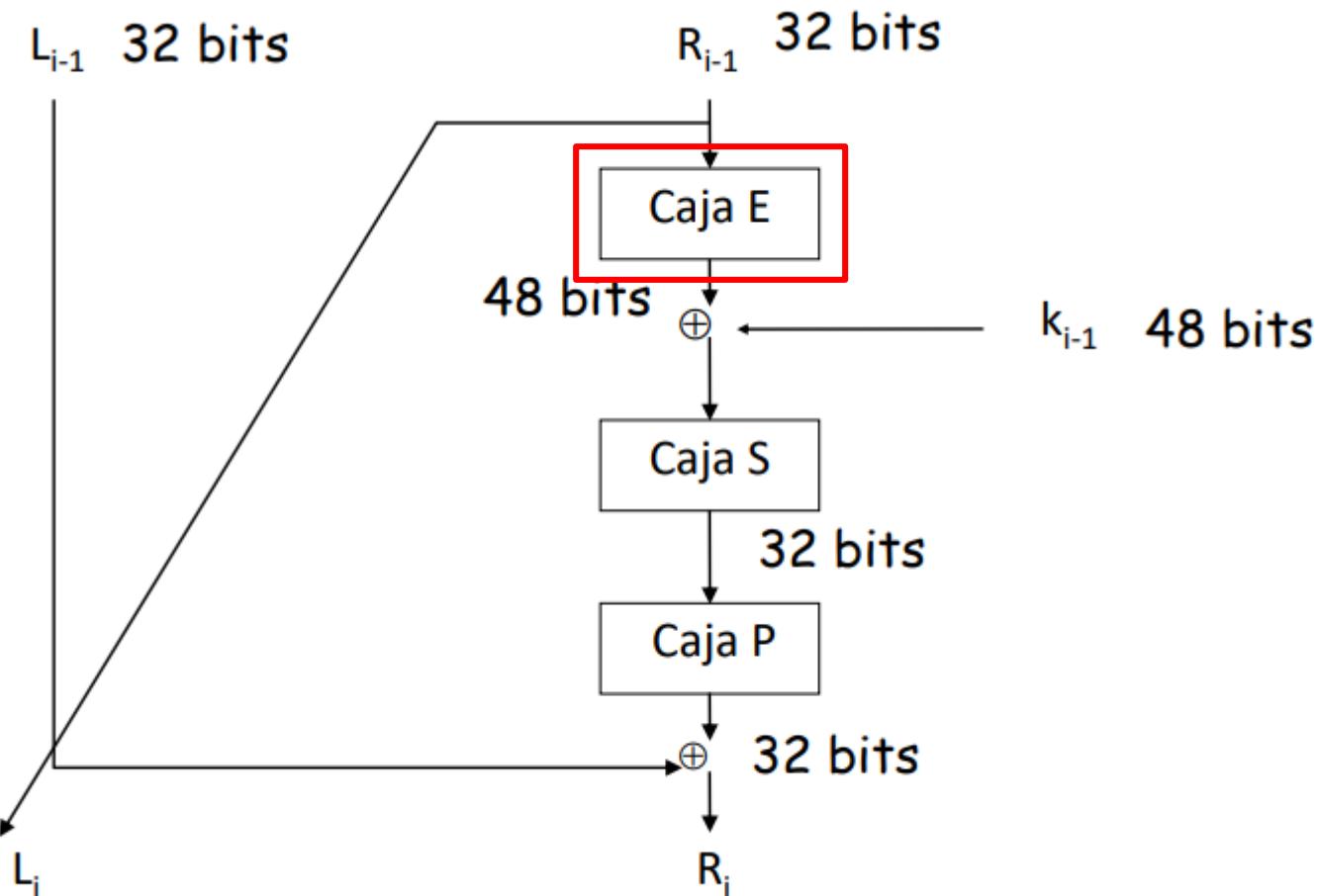


# DES: Expansion (E) Box

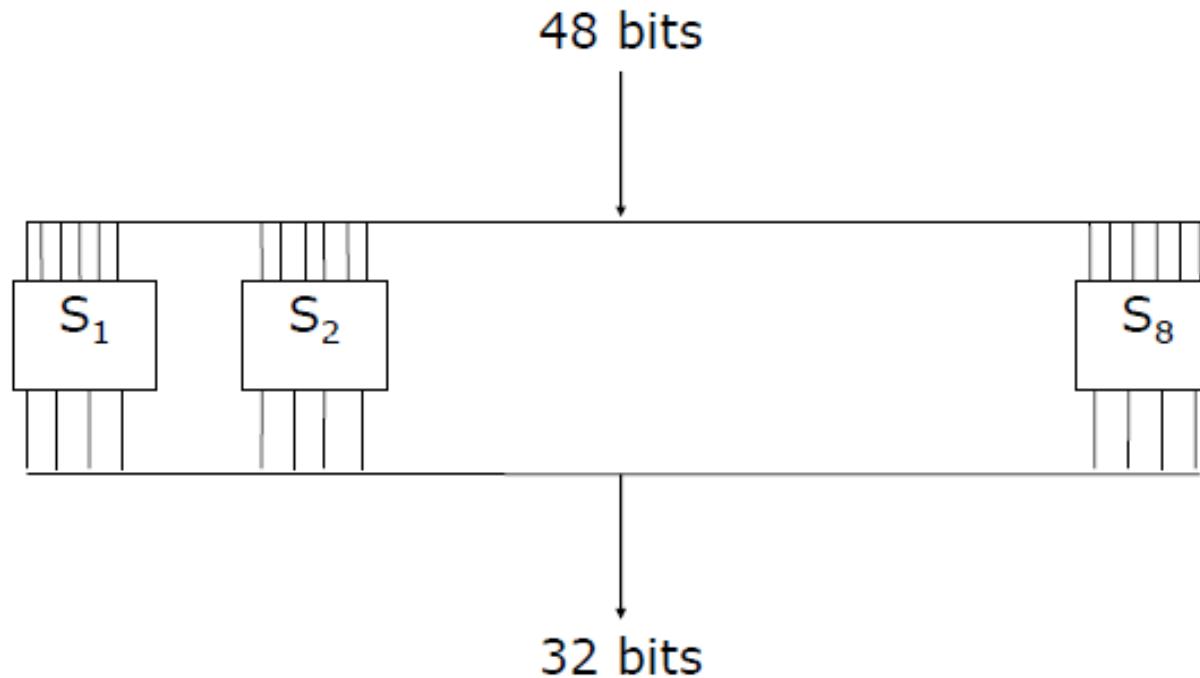
- From 32 bits to 48 bits

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

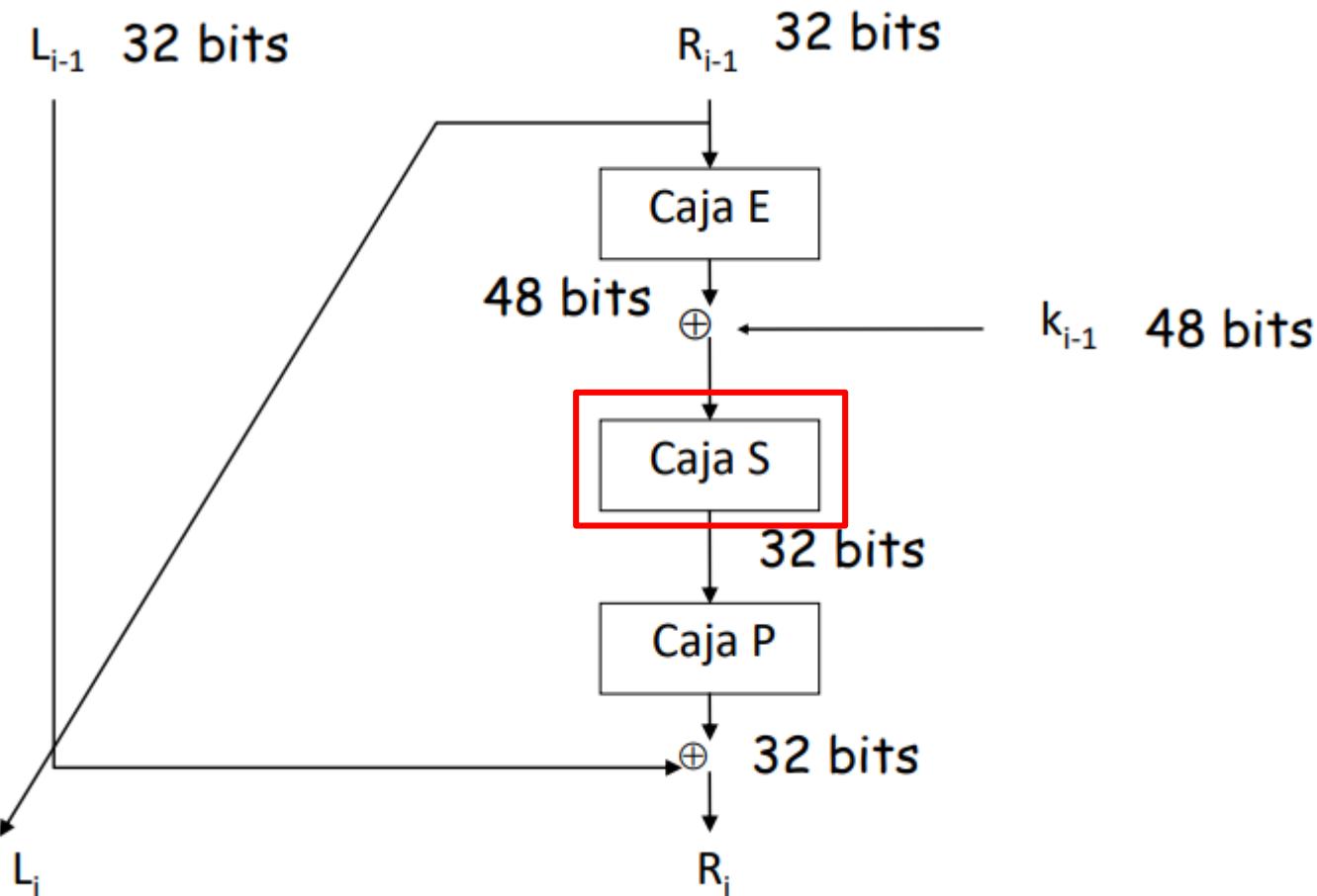
# DES: “Situational map” - Round



# DES: S boxes



# DES: “Situational map” - Round



# DES: S boxes

- **8 matrices  $S_1, S_2 \dots S_8$ , where each  $S_i$  receives a 6-bit input  $b_0, b_1 \dots b_5$**

Nº	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
2	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
3	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

- The decimal value of bit pair  $b_0 b_5$  select a row in matrix  $S_i$
- The decimal value of bits  $b_1 b_2 b_3 b_4$  select a column in  $S_i$
- The output in decimal is the value in the matrix given by the previously computed (row, column). Its 4-bit binary value is the output
- Eg:  $S_1$  & input= 110011 -> row 3 (11) and column 9 (1001), with a decimal output of, 11, that corresponds to the binary string 1011

# DES: S boxes

**S<sub>1</sub>**

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
2	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
3	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

**S<sub>2</sub>**

15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9

**S<sub>3</sub>**

10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12

**S<sub>4</sub>**

7	13	14	3	0	6	9	10	1	2	8	5	11	12	4	15
13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
10	6	9	0	12	11	7	13	15	1	3	14	5	2	8	4
3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14

# DES: S boxes

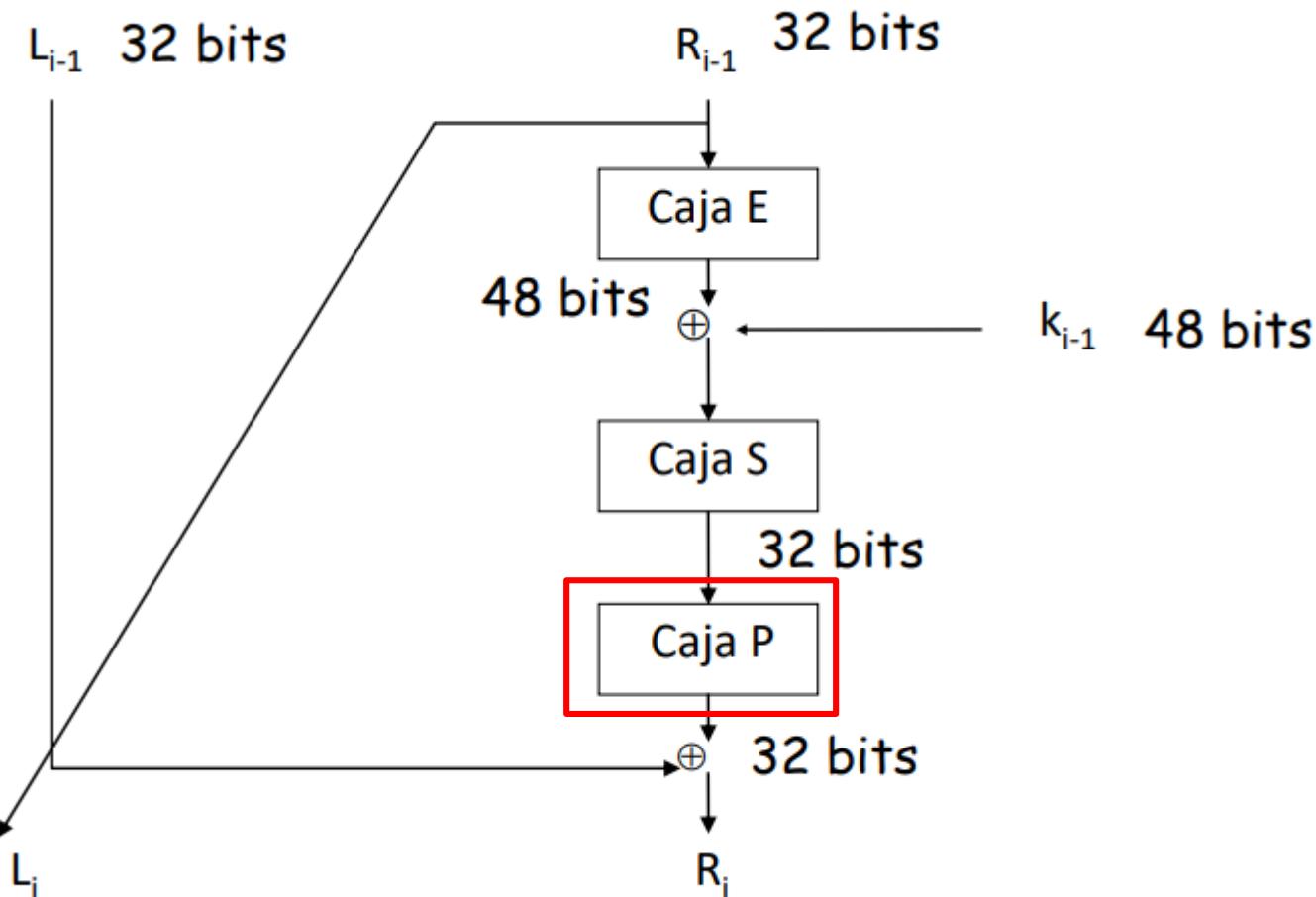
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
<b>S<sub>5</sub></b>	2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
	14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
	4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
	11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3
<b>S<sub>6</sub></b>	12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
	10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
	9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
	4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13
<b>S<sub>7</sub></b>	4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
	13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
	1	4	11	13	12	3	7	14	10	15	6	8	0	5	9	2
	6	11	13	8	1	4	10	7	9	5	0	15	14	2	3	12
<b>S<sub>8</sub></b>	13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
	1	15	13	8	10	3	7	4	12	5	6	11	0	14	9	2
	7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
	2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11

# DES: Permutation (P) Box

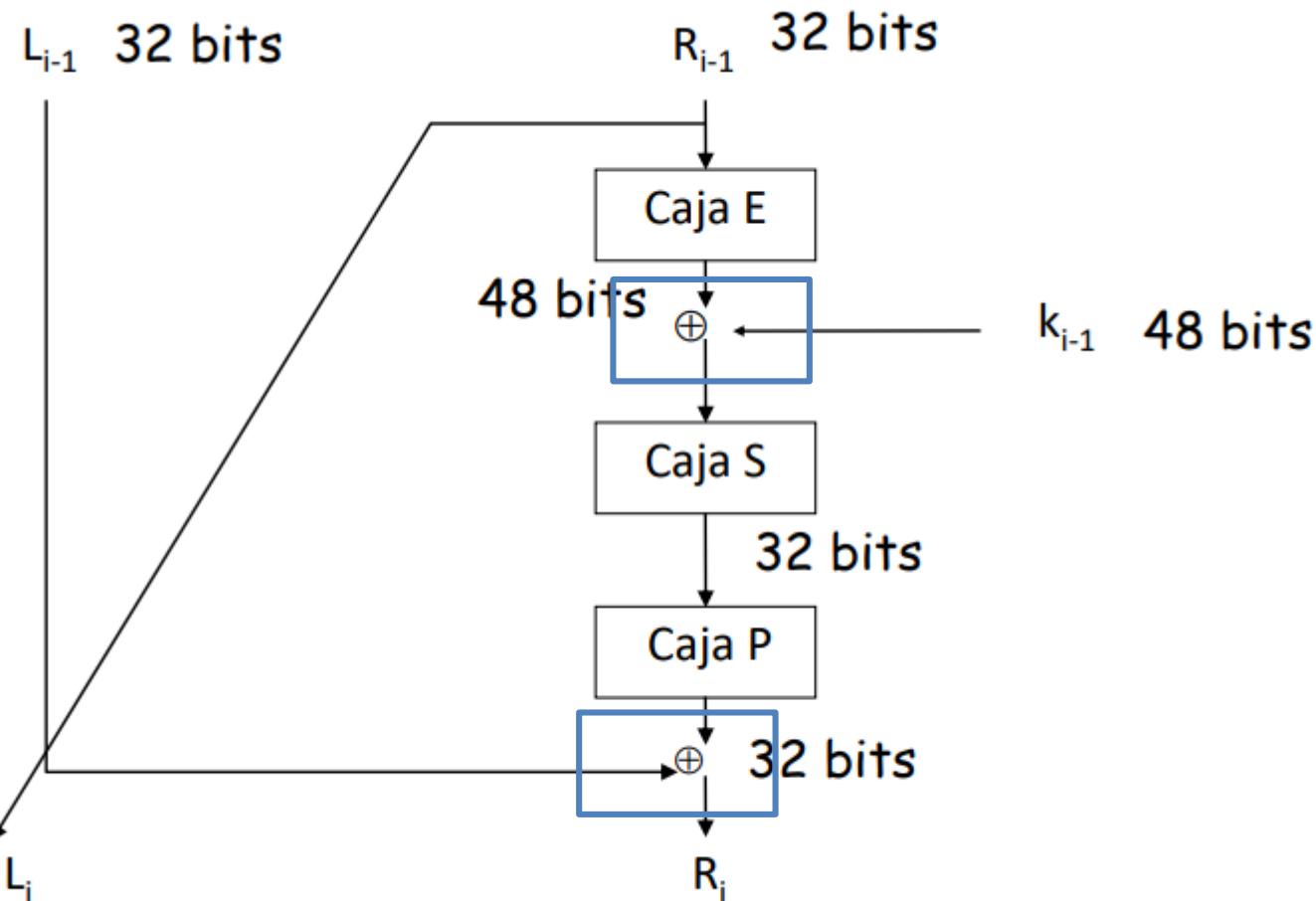
- From 32 bits to 32 bits

16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

# DES: “Situational map” - Round



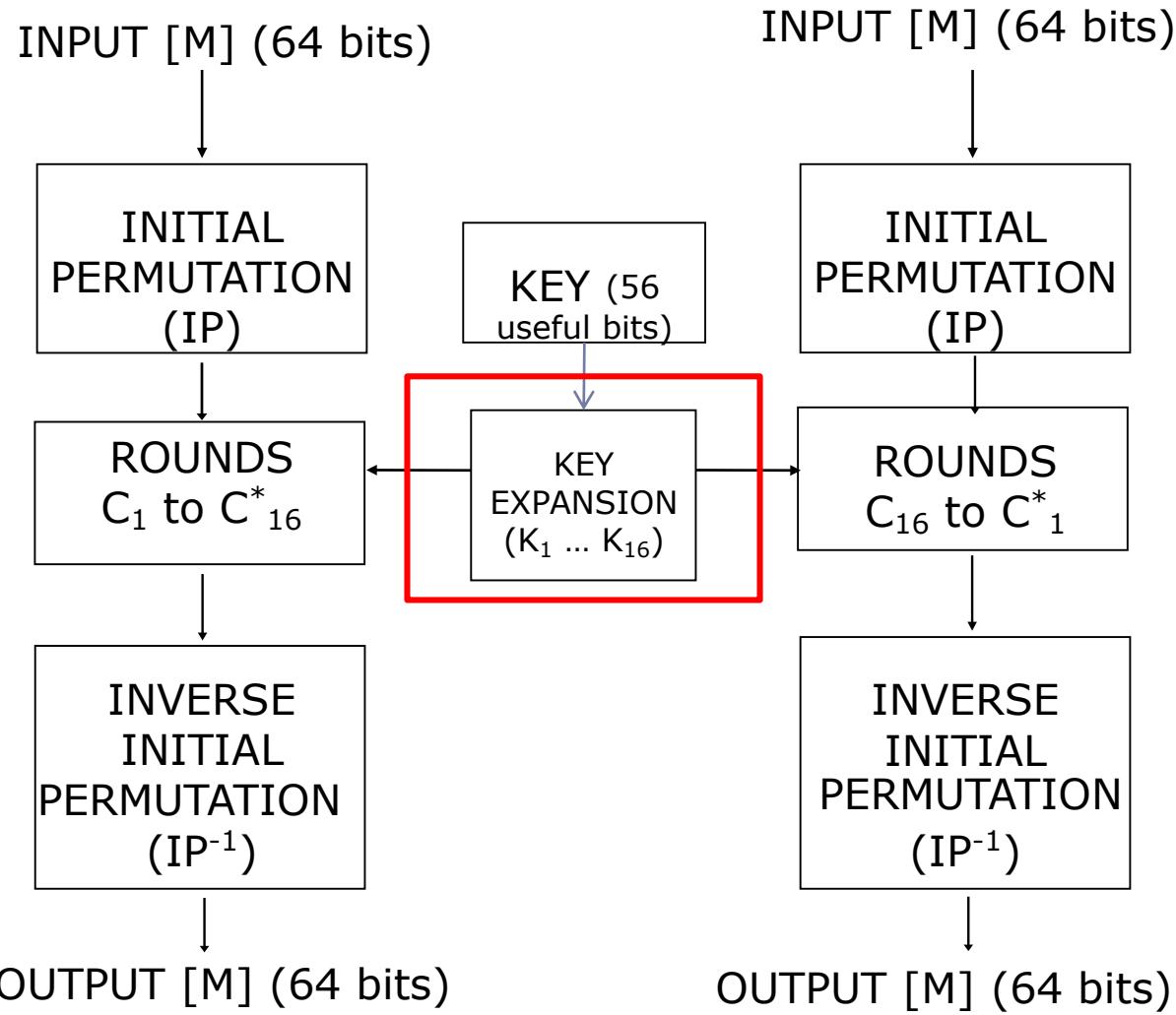
# DES: “Situational map” - Round



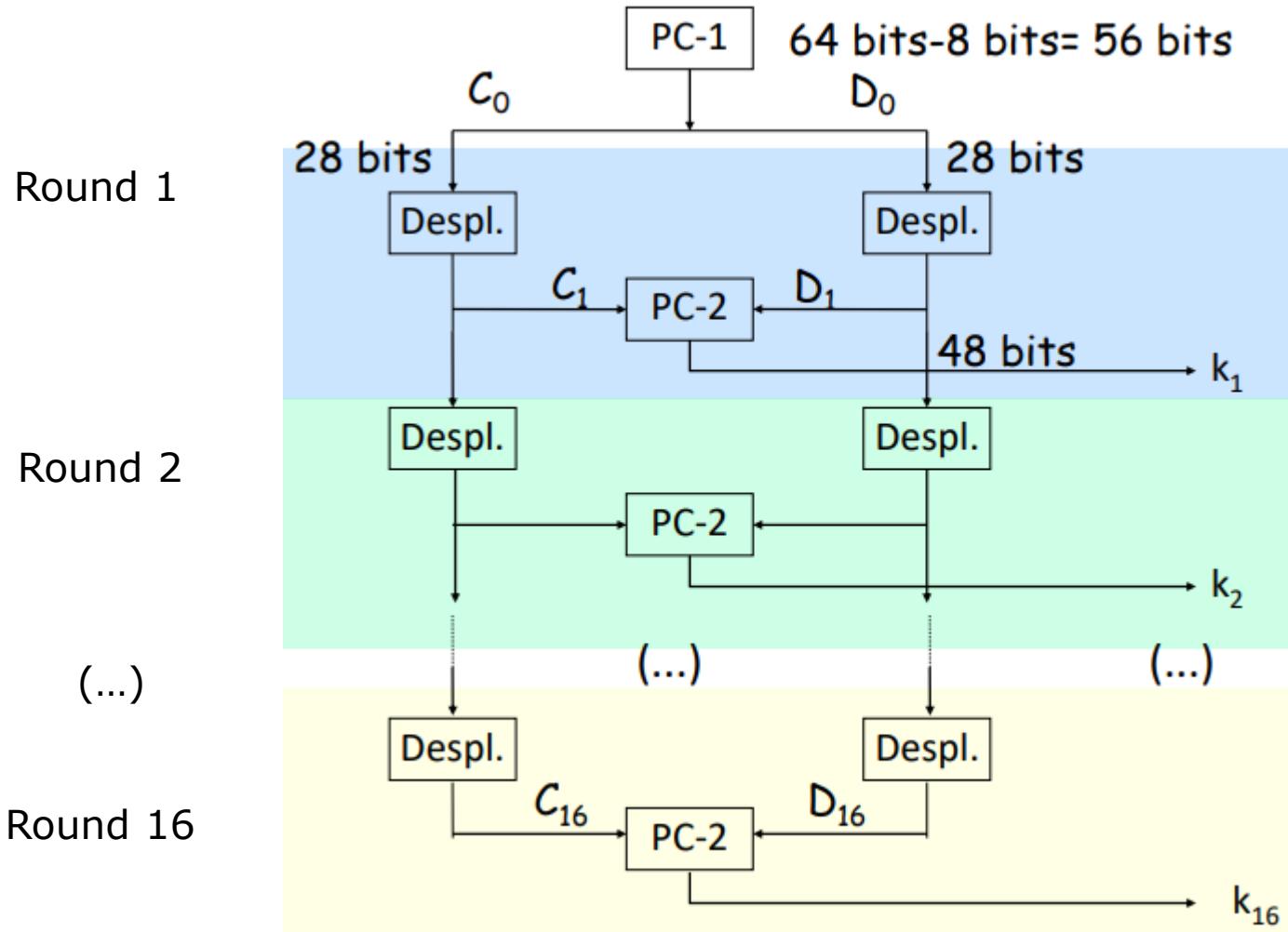
# OUTLINE

- (...)
  - Data Encryption Standard (DES)
    - Encryption
    - Key expansion
    - Decryption
    - Triple DES
    - Security
  - AES
  - (...)

# DES: “Situational map”



# DES: Generation of internal keys



# DES: Generation of internal keys

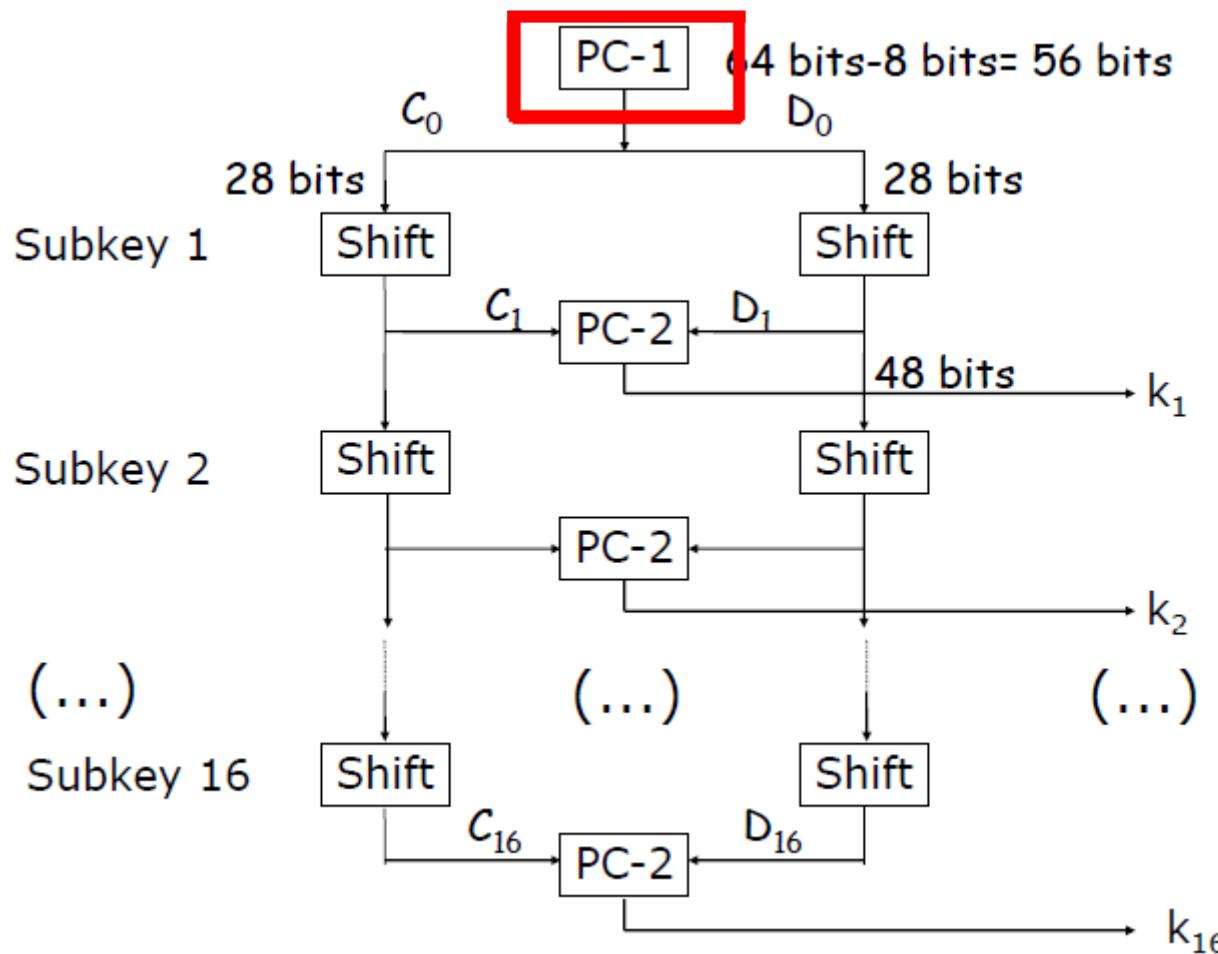
1. Permutation of the key PC-1 => 56 bit.
2. Divide block in 28 bits.
3.  $i=1$
4. Left shift of each block (1 or 2 bits depending on the round)
5. Internal key  $K_i$  generation
  1. Concatenation of the 2 blocks => 56 bits
  2. Permutation PC-2 => 48 bit = internal key  $k_i$
  3.  $i = i + 1$
  4. Go back to 4 while  $i \leq 16$
6. Result: 16 internal keys, 48 bits each of them
7. In the encryption process  $K_1-K_{16}$  (in the decryption, the inverse order  $K_{16}-K_1$ )

# DES: Permutation PC-1

- From 64 bits to 56 bits

57	49	41	33	25	17	9
1	58	50	42	34	26	18
10	2	59	51	43	35	27
19	11	3	60	52	44	36
63	55	47	39	31	23	15
7	62	54	46	38	30	22
14	6	61	53	45	37	29
21	13	5	28	20	12	4

# DES: “Situational map” – internal keys



# DES: Permutation PC-1

- Division in two halves,  $C_0$  y  $D_0$

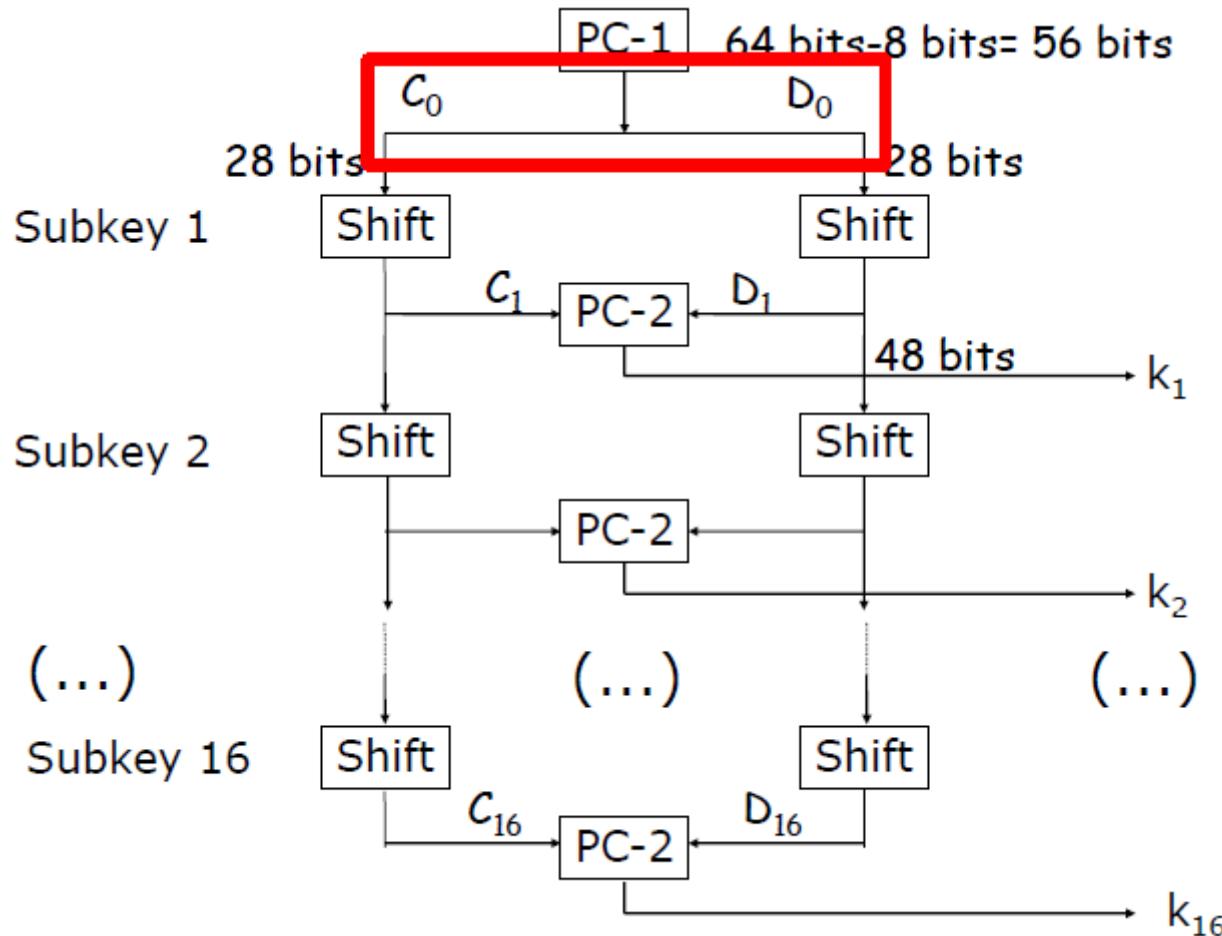
57	49	41	33	25	17	9
1	58	50	42	34	26	18
10	2	59	51	43	35	27
19	11	3	60	52	44	36

Block  $C_0$

63	55	47	39	31	23	15
7	62	54	46	38	30	22
14	6	61	53	45	37	29
21	13	5	28	20	12	4

Block  $D_0$

# DES: “Situational map” – internal keys



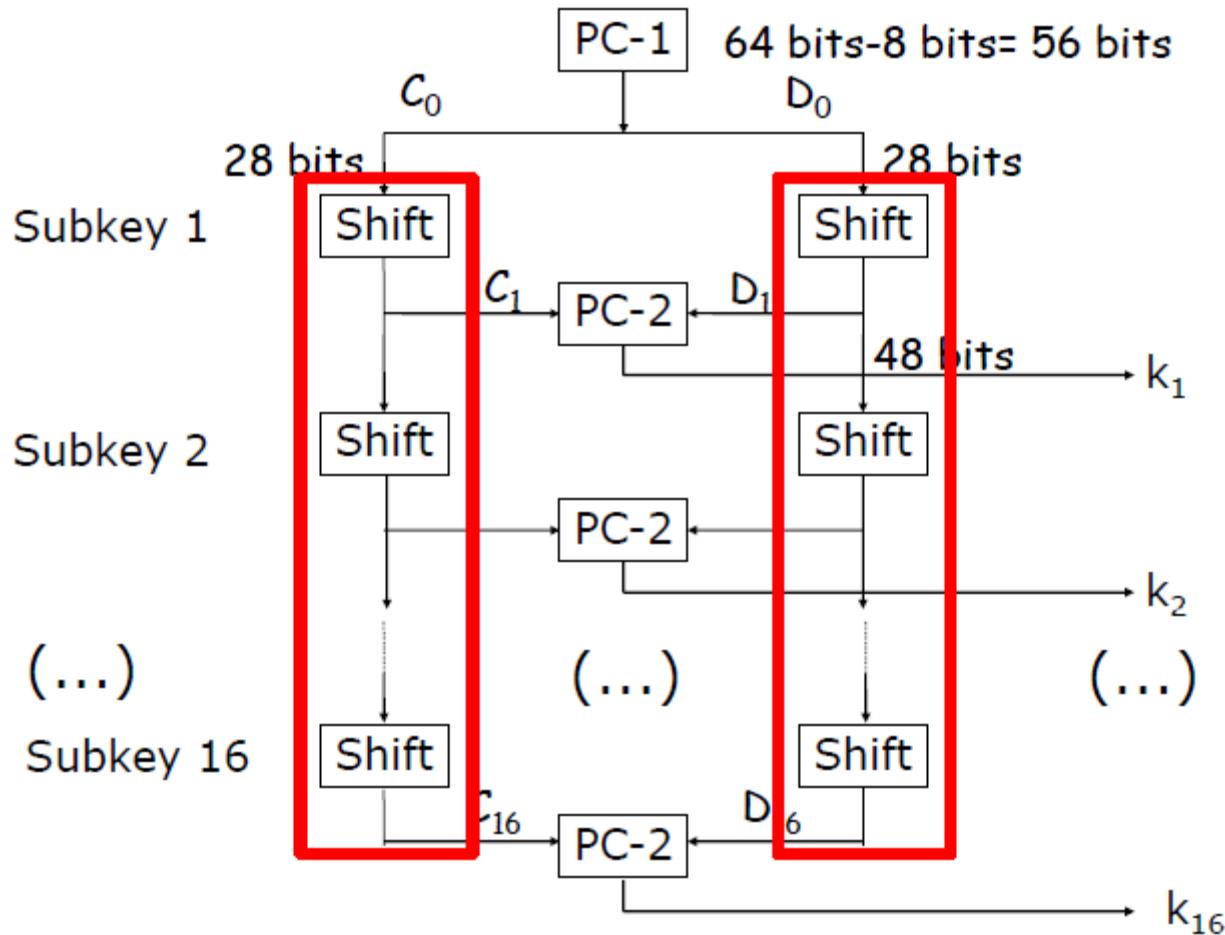
# DES: Shift

- Left circular shift within each of the halves

**Subkey nº** [ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 ]

**Bits rotated** [ 1 1 2 2 2 2 2 2 1 2 2 2 2 2 2 1 ]

# DES: “Situational map” – internal keys

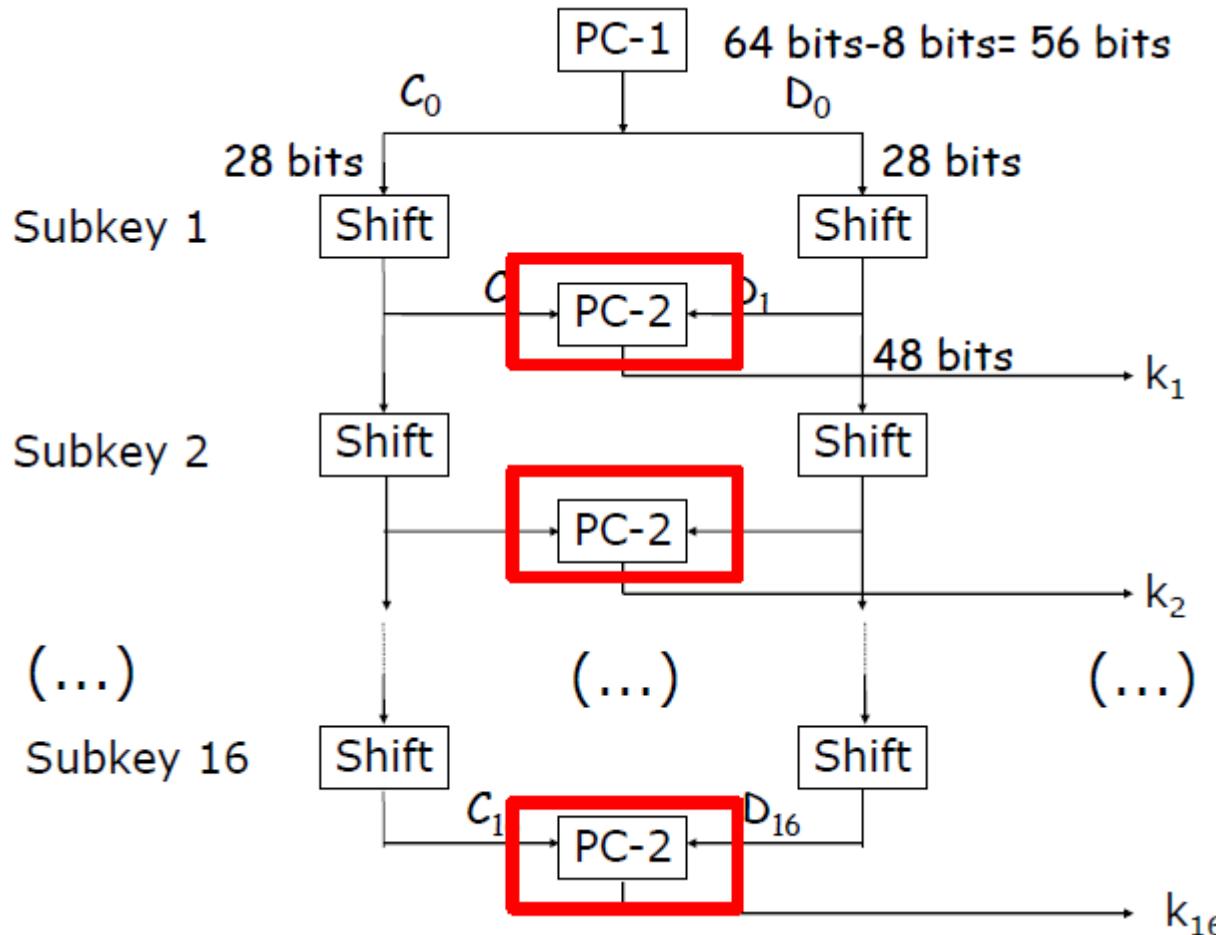


# DES: Permutation PC-2

- From 56 bits to 48 bits

14	17	11	24	1	5
3	28	15	6	21	10
23	19	12	4	26	8
16	7	27	20	13	2
41	52	31	37	47	55
30	40	51	45	33	48
44	49	39	56	34	53
46	42	50	36	29	32

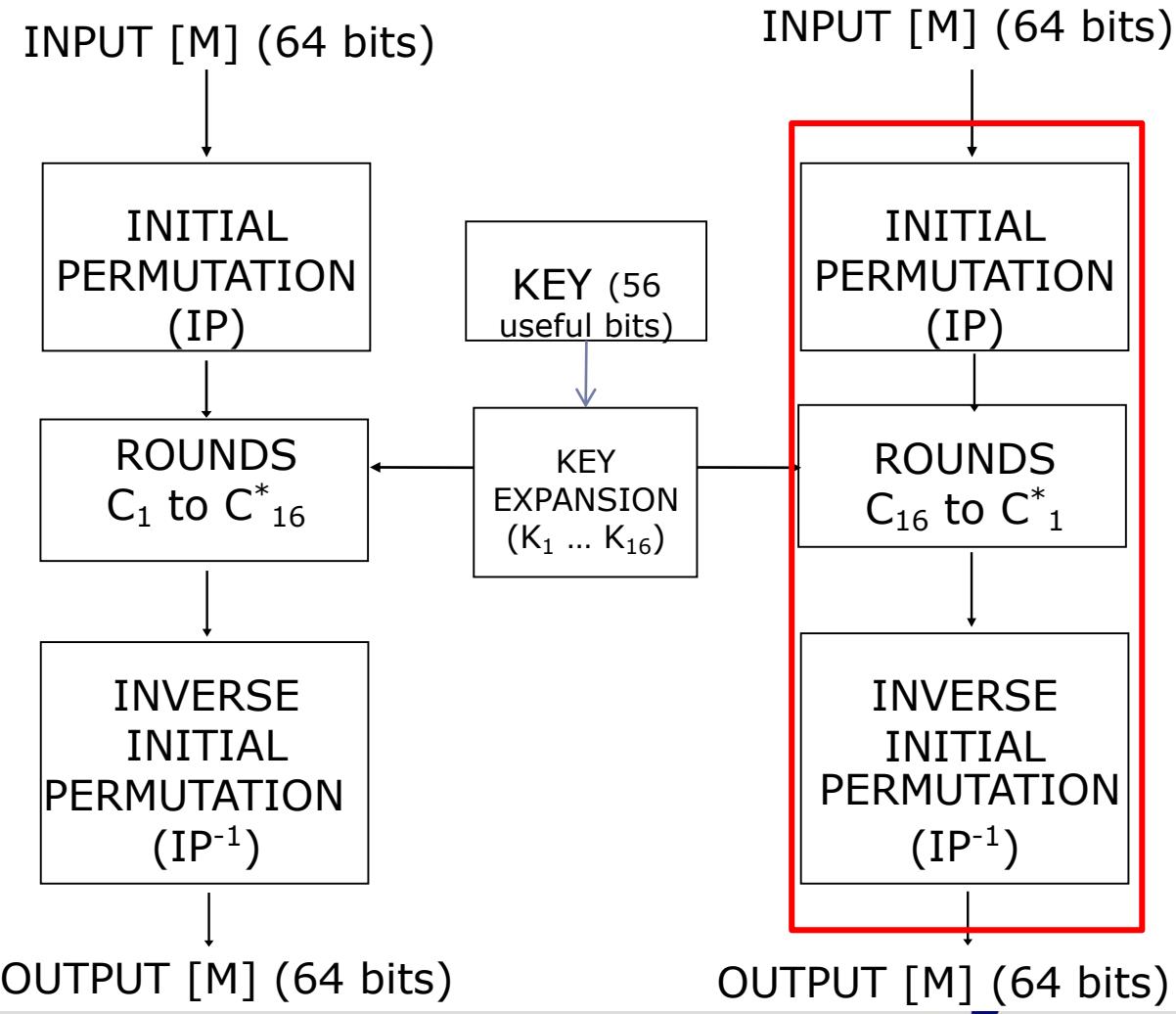
# DES: “Situational map” – internal keys



# OUTLINE

- (...)
  - Data Encryption Standard (DES)
    - Encryption
    - Key expansion
    - **Decryption**
    - Triple DES
    - Security
  - AES
  - (...)

# DES: “Situational map”



# DES: Decryption

- Same algorithms but with 2 changes
  - Internal keys used in inverse order
- The key expansion algorithm is the same, but it should be computed “going up” instead of “going down”
  - After PC-1:  $C_0=C_{16}$  and  $D_0=D_{16}$
  - Shift to  $C_i$  and  $D_i$  should be “right shifts”

# OUTLINE

- (...)
  - Data Encryption Standard (DES)
    - Encryption
    - Key expansion
    - Decryption
    - **Triple DES**
    - Security
  - AES
  - (...)

# Triple DES (TDES)

- 3 DES con 2 claves => clave de 112 bit
  - $C = E(k_1, D(k_2, E(k_1, M)))$
  - Compatibility with simple DES if  $k_1=k_2$
- 3 DES con 3 claves => clave de 112 bit
  - $C = E(k_3, D(k_2, E(k_1, M)))$
- Cost of the meet in the middle attack  $2^{112}$

# OUTLINE

- (...)
  - Data Encryption Standard (DES)
    - Encryption
    - Key expansion
    - Decryption
    - Triple DES
    - Security
  - AES
  - (...)

# Security

- Attacks to DES
  - Brute force
    - Broken in less than one day  
<http://www.scieengines.com/company/news-a-events/74-des-in-1-day.html>
  - Differential cryptoanalysis (Biham and Shamir)
    - $2^{47}$  chosen plaintexts needed. Effort on  $2^{47}$  encryptions
    - Lucifer was vulnerable but DES is not
  - Linear Cryptanalysis (Matsui)
    - $2^{43}$  knownplaintexts needed
- Attack to Triple DES
  - Meet-in-the-middle attack
    - Reduces the effort to an order of 256

# OUTLINE

- 5. Symmetric encryption: Block ciphers
  - Modern encryption
  - Block ciphers
    - Introduction
    - Feistel scheme
    - Operation modes
    - Block ciphers: advantages and disadvantages
    - DES
    - AES

# Advanced Standard Encryption (AES)

- Advanced Encryption Standard
- Standard for symmetric encryption (block cipher)
- NIST contest to substitute DES
  - Government communications
  - Bank transfers
  - Electronic commerce
  - Etc.

# Advanced Standard Encryption (AES)

- At least as secure as 3DES but faster in SW
- Symmetric block cipher
- Block size: 128 (16 bytes)
- Key sizes: 128, 192, 256

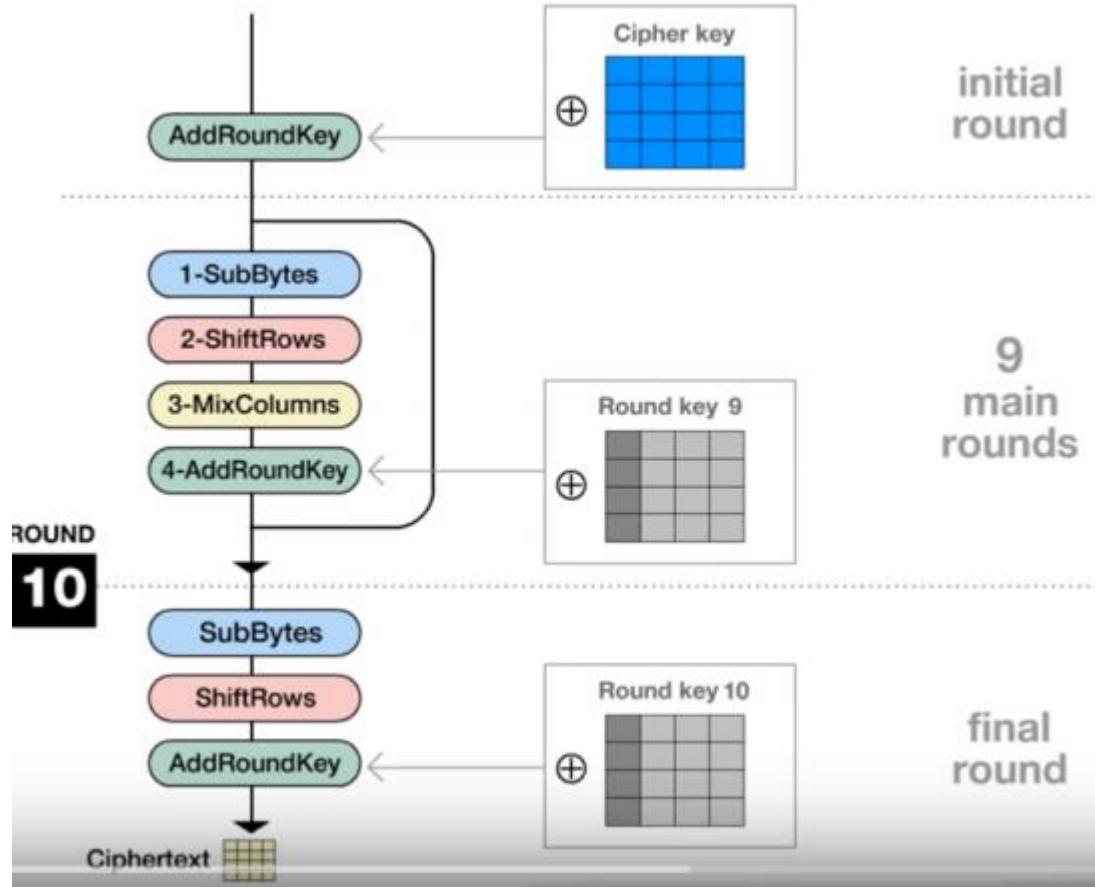
# AES: Evaluation criteria

<b>Security</b>	<b>Cost</b>	<b>Features</b>
<ul style="list-style-type: none"><li>• Compared to the other proposals</li><li>• Output indistinguishable from a random permutation</li><li>• Sound Mathematical foundations</li><li>• Resistance against known cryptanalytical attacks</li></ul>	<ul style="list-style-type: none"><li>• No royalties</li><li>• Efficiency – both in HW and SW</li><li>• Memory requirements</li></ul>	<ul style="list-style-type: none"><li>• Flexibility Able to manage different block and key sizes</li><li>• Simplicity</li></ul>

# AES: Winner features

- Operates on blocks of 16 bytes (128 bits)
- Accepts 3 key sizes 128, 192, 256 bits
- Substitución-permutation network (not a Feistel network)
- Fast in SW and HW, easy to implement and low memory requirements
- Based on 4 reversible functions, applied n rounds
- State matrix evolution

# AES scheme



- Encryption functions (from the top to the bottom):
  - *AddRoundKey*
  - *ByteSub*
  - *ShiftRow*
  - *MixColumns*
- Decryption functions (from the bottom to the top):
  - *InvAddRoundKey*
  - *InvByteSub*
  - *InvShiftRow*
  - *InvMixColumns*

Author: Enrique Zabala

<http://www.formaestudio.com/rijndaelinspector/>

# AES: key bytes and states

- Key bytes and state bytes are placed in rectangular arrays

k 0,0	k 0,1	k 0,2	k 0,3	k 0,4	k 0,5	k 0,6	k 0,7
k 1,0	k 1,1	k 1,2	k 1,3	k 1,4	k 1,5	k 1,6	k 1,7
k 2,0	k 2,1	k 2,2	k 2,3	k 2,4	k 2,5	k 2,6	k 2,7
k 3,0	k 3,1	k 3,2	k 3,3	k 3,4	k 3,5	k 3,6	k 3,7

Variable block size:  
16, 24 or 32 bytes  
AES: 16 bytes

Key sizes:  
16, 24 or 32 bytes

a 0,0	a 0,1	a 0,2	a 0,3	a 0,4	a 0,5	a 0,6	a 0,7
a 1,0	a 1,1	a 1,2	a 1,3	a 1,4	a 1,5	a 1,6	a 1,7
a 2,0	a 2,1	a 2,2	a 2,3	a 2,4	a 2,5	a 2,6	a 2,7
a 3,0	a 3,1	a 3,2	a 3,3	a 3,4	a 3,5	a 3,6	a 3,7

# RINDAEL algorithm

```
Rijndael(State, Key) {
```

```
    KeyExpansion( Key, ExpandedKey );  
    AddRoundKey( State, ExpandedKey );  
    for (i=1; i<10; i++)  
        Round(State, ExpandedKey+4);  
    FinalRound(State, ExpandedKey+4X10);
```

```
}
```

```
Round(State, RoundKey) {
```

```
    ByteSub(State);  
    ShiftRow(State);  
    MixColumn(State);  
    AddRoundKey(State,  
    RoundKey);
```

```
}
```

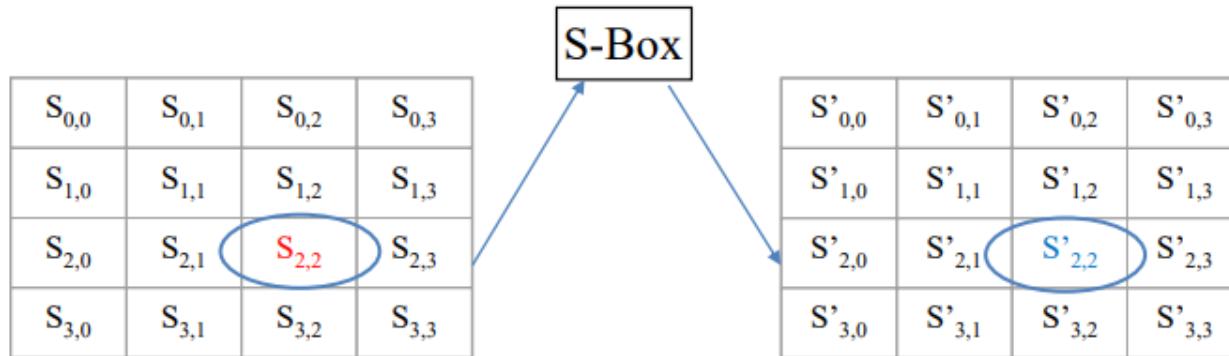
<b>State</b>	-- array of 4 words (de 32 bits)
<b>No. of Rounds</b>	-- 9 rounds
<b>KeyExpansion</b>	-- XOR of the keywords, S-box lookups, rotation of bytes intra-word
<b>AddRoundKey</b>	-- bitwise-XOR with the keywords
<b>FinalRound</b>	-- similar to a Round but without MixColumn

# AES: Operation with bytes

- Additions and multiplications: Galois field GF(2<sup>8</sup>) with 8 bits
- The following polynomial is used:

$$p(x) = x^8 + x^4 + x^3 + x + 1.$$

# AES: SubByte function



$$\begin{pmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

Inverse of the input

Value {63}<sub>16</sub> or {011000011}<sub>2</sub>

# AES: SubByte table

- The inverse of the input can be also computed through this table
- 5a => be

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

# AES: SubByte example

- Calculus of SubByte(5a)

$$5a = 01011010 = x^6 + x^4 + x^3 + x + 1$$

$$\text{inv}(5A) = 22 = 00100010$$

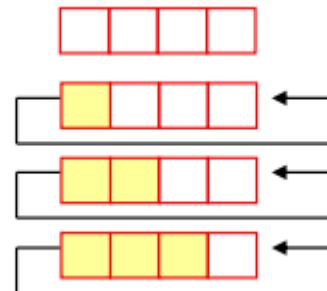
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

- Once operations are performed, the result is:  $1011\ 1110 = \text{be}$  (the same as using the table)

# AES: Shiftrows function

- Row 0 => no swift
- Row 1 => Swift 1 byte
- Row 2 => Swift 2 bytes
- Row 3 => Swift 3 bytes

$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$
$S_{1,0}$	$S_{1,1}$	$S_{1,2}$	$S_{1,3}$
$S_{2,0}$	$S_{2,1}$	$S_{2,2}$	$S_{2,3}$
$S_{3,0}$	$S_{3,1}$	$S_{3,2}$	$S_{3,3}$



$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$
$S_{1,1}$	$S_{1,2}$	$S_{1,3}$	$S_{1,0}$
$S_{2,2}$	$S_{2,3}$	$S_{2,0}$	$S_{2,1}$
$S_{3,3}$	$S_{3,0}$	$S_{3,1}$	$S_{3,2}$

# AES: Mixcolumns function

- We work in GF(28). Polynomial  $p(x) = x^8 + x^4 + x^3 + x + 1$ .
- Remember:  $\{03\} = x + 1$ ,  $\{02\} = x$ ,  $\{01\} = 1$ .

$$\begin{bmatrix} S'_{0,C} \\ S'_{1,C} \\ S'_{2,C} \\ S'_{3,C} \end{bmatrix} = \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \begin{bmatrix} S_{0,C} \\ S_{1,C} \\ S_{2,C} \\ S_{3,C} \end{bmatrix}$$

- Operations over each column:

$$S'_{0,C} = (\{02\} \bullet S_{0,C}) \oplus (\{03\} \bullet S_{1,C}) \oplus S_{2,C} \oplus S_{3,C}$$

$$S'_{1,C} = S_{0,C} \oplus (\{02\} \bullet S_{1,C}) \oplus (\{03\} \bullet S_{2,C}) \oplus S_{3,C}$$

$$S'_{2,C} = S_{0,C} \oplus S_{1,C} \oplus (\{02\} \bullet S_{2,C}) \oplus (\{03\} \bullet S_{3,C})$$

$$S'_{3,C} = (\{03\} \bullet S_{0,C}) \oplus S_{1,C} \oplus S_{2,C} \oplus (\{02\} \bullet S_{3,C})$$

# AES: Mixcolumns example

- We assume this is the intermediate state

e1	a8	63	0d
fb	18	f4	c8
96	5b	73	11
7c	a0	e6	fd

- The first byte of the state matrix ( $S'_{0,0}$ ) is:

$$S'_{0,0} = \{02\}S_{0,0} \oplus \{03\}S_{1,0} \oplus S_{2,0} \oplus S_{3,0}; S'_{0,0} = \{02\}e1 \oplus \{03\}fb \oplus 96 \oplus 7c$$

$$\{02\}e1 = x(x^7 + x^6 + x^5 + 1) = x^8 + x^7 + x^6 + x;$$

$$\{02\}e1 = (x^8 + x^7 + x^6 + x) \bmod (x^8 + x^4 + x^3 + x + 1) = d9$$

$$\{03\}fb = (x + 1)(x^7 + x^6 + x^5 + x^4 + x^3 + x + 1)$$

$$\{03\}fb = x^8 + x^3 + x^2 + 1$$

$$\{03\}fb = (x^8 + x^3 + x^2 + 1) \bmod (x^8 + x^4 + x^3 + x + 1) = 16$$

# AES: Mixcolumns example

- The first byte of the state matrix ( $S'_{0,0}$ ) is:

$$S'_{0,0} = \{02\}S_{0,0} \oplus \{03\}S_{1,0} \oplus S_{2,0} \oplus S_{3,0}; S'_{0,0} = \{02\}\text{e1} \oplus \{03\}\text{fb} \oplus \text{96} \oplus \text{7c}$$
$$\{02\}\text{e1} = (0000\ 0010)(1110\ 0001) = (1\ 1100\ 0010);$$

$$\{02\}\text{e1} = (1\ 1100\ 0010) \oplus (1\ 0001\ 1011) = (1101\ 1000) = \text{d9}$$

$$\{03\}\text{fb} = (0000\ 0011)(1111\ 1011) = (0000\ 0010)(1111\ 1011) \oplus (1111\ 1011)$$

$$\{02\}\text{fb} = (0000\ 0010)(1111\ 1011) = (1\ 1111\ 0110)$$

$$\{02\}\text{fb} = (1\ 1111\ 0110) \oplus (1\ 0001\ 1011) = (1110\ 1101)$$

$$\{03\}\text{fb} = (1110\ 1101) \oplus (1111\ 1011) = (0001\ 0110) = \text{16}$$

$$S'_{0,0} = \text{d9} \oplus \text{16} \oplus \text{96} \oplus \text{7c}$$

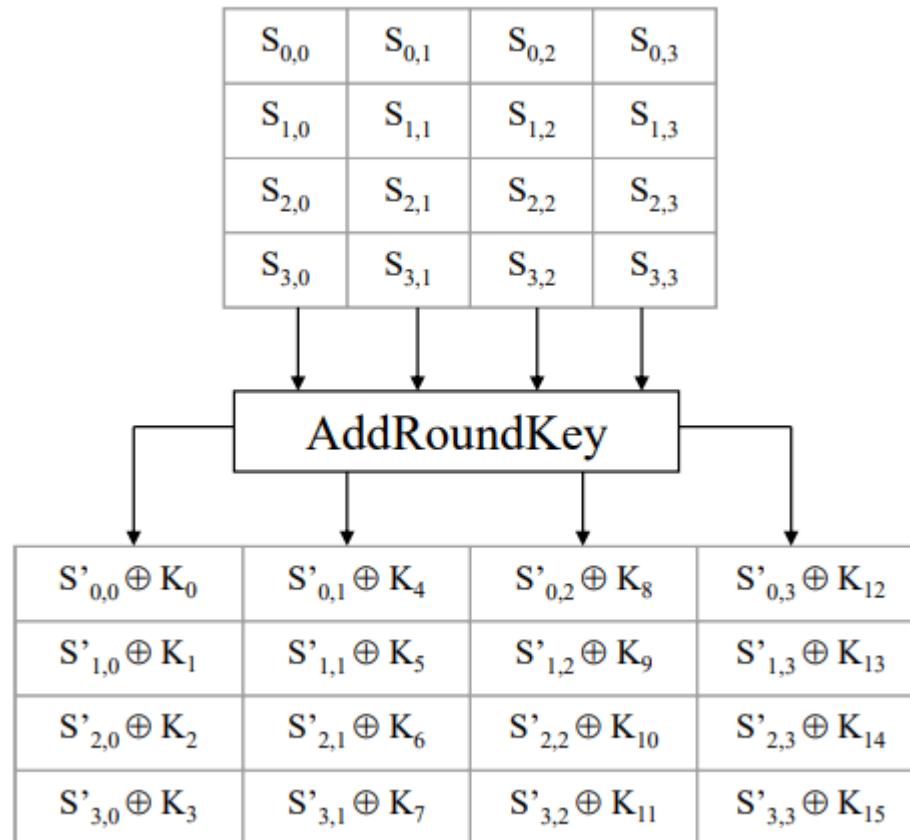
$$\text{Then: } S'_{0,0} = \text{25}$$

Similar calculations are computed until byte  $S'_{4,4}$

# AES: AddRoundKey function

AddRoundKey, XOR between the State and the round subkey.

Goal – round function does not depend on the key



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