



Classic cryptography

Proposed exercises

Note. In these exercises, consider the Spanish alphabet (that is, including 'ñ' between 'n' and 'o', 27 symbols) unless otherwise stated.

Exercise 1 :

Considering the encryption function $E(m)=7m+3 \pmod{27}$, answer the following questions

- Which are the values of the decimation and shift constants?
- Encrypt "TERCERA"
- Decrypt "DID ÑOE"

Key:

- Decimation constant = 7; Shift constant =3
-

$$E("T") = E(20) = 20 \cdot 7 + 3 \pmod{27} = 8 = "I"$$

$$E("E") = E(4) = 4 \cdot 7 + 3 \pmod{27} = 31 \pmod{27} = 4 = "E"$$

This process is repeated until getting the final message: "IEUQ EUD"

- First we get the decryption equation:

$7^{-1} \pmod{27} = 4$. Thus, the decryption equation is as follows

$$D(c) = 4(c - 3) \pmod{27} = 4c - 12 \pmod{27} = 4c + 15 \pmod{27}$$

$$D("D") = E(3) = 4 \cdot 3 + 15 \pmod{27} = 0 = "A"$$

$$D("I") = E(8) = 4 \cdot 8 + 15 \pmod{27} = 20 = "T"$$

This process is repeated until getting the final message: "ATAQUE"

Exercise 2:

Given the key "LUCI" encrypt the message M= "CAMINERO" using Vigenère.

Key:

NUÑPXYTW

Exercise 3:

Given the key "PLUS" decrypt the message C= "LSAW COMW" given that it was encrypted using Vigenère.

Key:

VIGENERE

Exercise 4:

Given the key "ALA" decrypt the message C= "EDVI KVQG" given that it was encrypted using Vigenère with autokey

Key:

Given that we are dealing with the autokey variant, it is needed to decrypt it step by step as follows:

EDVI KVQG

ALA

ESV

EDVI KVQG

ALAE SV

ESVE RA

EDVI KVQG

ALAE SV ER

ESVE RANO

Exercise 5:

Given the key "MARTES", encrypt M= "FALSO PUENTE" using Playfair

Key:

BE GF PQ ZF QM RZ

Consider that the matrix is the following one:

M A R T E

S B C D F

G H I/J K L

N/Ñ O P Q U

V W X Y Z

Exercise 6:

Given the key "MARTES" decrypt C= "FOMUMB ZFTERZ" given that it was encrypted using Playfair

Key:

BUENA SUERTE X

The matrix is the same is in the previous exercise.

Exercise 7:

Given the matrix $K = \begin{bmatrix} 3 & 2 \\ 4 & 6 \end{bmatrix}$ answer the following questions:

- Determine if it is suitable as key for Hill ciphers.
- Encrypt M="RECORDAR" using Hill cipher.

Key:

- $\det(K)=10 \neq 0$ and $\gcd(\det(K), 27)=1$, this is suitable to be used in Hill.
- We need to work on the message in pairs of characters as follows:

$$\text{Encrypt (RE)} = \begin{bmatrix} 3 & 2 \\ 4 & 6 \end{bmatrix} * \begin{pmatrix} 18 \\ 4 \end{pmatrix} = (62 \ 96) \bmod 27 = (8 \ 15) = \text{"IO"}$$

Repeating the process for the remaining pairs, it leads to: C="IOJQ GJJA"

Exercise 8:

Given the matrix $K = \begin{bmatrix} 7 & 6 \\ 3 & 11 \end{bmatrix}$ answer the following question:

- Decrypt C="J8D6" considering the English alphabet with numbers in the following order {A,...,Z}+{0,...,9}..

Key:

- First, we need to compute the inverse matrix:

Given that $\text{Det}(K) = |K| = 23$, and that $\gcd(23,36)=1$, we can compute the inverse $\text{Det}(K)^{-1} = |K|^{-1} = 11$. This result can be used to compute the inverse of the matrix,

$$K^{-1} = |K|^{-1} \cdot \text{adj}(A)^T \bmod 36 = \begin{bmatrix} 13 & 6 \\ 3 & 5 \end{bmatrix}$$

Now we operate the ciphertext in pairs, as follows:

$$K^{-1} \cdot \text{"J8"} \bmod 36 = \begin{bmatrix} 13 & 6 \\ 3 & 5 \end{bmatrix} * \begin{bmatrix} 9 \\ 34 \end{bmatrix} = \begin{bmatrix} 33 \\ 17 \end{bmatrix} = \text{"7R"}$$

We repeat the process for "D6". The final result is: "7RPZ"

Exercise 9:

Consider the permutation $K_p = (642135)$. Decrypt the message $C = \text{"OOEMTD IACSLS EEOCSE"}$ which has been encrypted using that permutation.

Key:

OOEMTD IACSLS EEOCSE

1 234 5 6 123456 123456 → Re-ordering based on the permutation:

$M = \text{"METODOS CLASICOS"}$

Exercise 10:

Encrypt the following message $M = \text{"FIESTA NACIONAL"}$ using a 4-column transposition

Key:

It should be transposed as follows:

F I E S
T A N A
C I O N
A L X X

Thus, the result is $C = \text{"FTCAI AILENO XSANX"}$ (that is, reading the matrix by columns)