Problem Set 7
Heteroskedasticity

Note: In those exercises based on estimations in which the data used is referred, the student should check in Gretl the presented results.

1. Consider two independent random variables Y_1 and Y_2, with $E(Y_1) = E(Y_2) = \mu$ and $V(Y_1) = \sigma^2, V(Y_2) = 4\sigma^2$. Given two observaciones (y_1, y_2), we consider two alternative estimators of the unknown parameter μ :

$$\begin{aligned} m &= \frac{1}{2}y_1 + \frac{1}{2}y_2 \\ m' &= \frac{3}{5}y_1 + \frac{2}{5}y_2. \end{aligned}$$

   a) Check that both m and m' are unbiased estimators and calculate their corresponding variances. What estimator would you choose?

   b) Obtain the unbiased estimator $m^* = cy_1 + (1-c)\, y_2$ with the minimum variance. What is the relation of the weight ratio of each observation, $c/(1-c)$, with the corresponding variances?

   c) Suppose that you now the sample means m_1 and m_2, obtained from two random samples of the same population with sizes 100 and 25, respectively. What is the best estimator that you can get? (Hint: recall that under random sampling, the variance of a sample mean is inversely proportional to the sample size, and that observations of the two samples are independent each other.)

2. [Based on problem 8.6 in Wooldridge textbook] Use SLEEP75 dataset to estimate the following equation for sleeping hours

$$sleep = \beta_0 + \beta_1 totwrk + \beta_2 educ + \beta_3 age + \beta_4 age^2 + \beta_5 yngkid + \beta_6 male + u.$$

   a) Test for heteroskedasticity using the White test in GRETL (menu *test*) in the model for a significance level of 10 %.

   b) Write a model allowing the variance of u to differ between men and women. The variance should not depend on further factors.

   c) Estimate the model parameters for heteroskedasticity. (You have to estimate first by OLS the equation for *sleep*, to obtain the OLS residuals) Is the estimated variance of u higher for men or for women?

   d) Does the variance of u differ significantly between men and women?

3. In a linear regression model in which all the usual assumption, except homoskedasticity, hold, explain whether each of the following statements is true or false:

   a) The OLS estimator is consistent.

   b) The conventional expression for the variance of the OLS estimator is incorrect.

c) Departure from the homoskedasticity assumption does not affect the properties of the OLS estimator, which continues being the Best Linear Unbiased Estimator.

4. The existence of heteroskedasticity is problematic since (indicate what, among the following statements, is correct):

   a) In its usual form, OLS assumes homoskedasticity and the parameter estimates rely on such assumption.

   b) In its usual form, OLS assumes homoskedasticity and the estimates of the parameters'variances rely on such assumption.

   c) The OLS estimator becomes biased.

   d) The estimate of the variance of the OLS estimator cannot be calculated.

5. If there is heteroskedasticity but it is ignored, and the usual OLS (without special options) command is used then (indicate what statements are correct):

   a) the parameter estimates will be inconsistent.

   b) the estimates of the variances of the parameters will be inconsitent.

   c) all the estimates of the variance of the parameters will be upward biased.

   d) None of the other answers is correct.

6. If we use OLS to estimate a linear regression model that satisfies homoskedasticity, but we use the calculate standard errors with the heteroskedasticity-robust option, then (indicate what statements are correct):

   a) We cannot make inference using such estimates, since the heteroskedasticity-robust standard errors are inconsistent under homoskedasticity.

   b) Under homoskedasticity, it would be better to use the usual calculation method for the standard errors. instead of the robust standar errors.

   c) Both under homoskedasticity and under heteroskedasticity, the heteroskedasticity-robust standard errors are consistent.

7. If estimate a heteroskedastic linear regression model by OLS, but using heteroskedasticity-robust standard errors (indicate what statement are correct):

   a) We will have the Best LInear Unbiased Estimator.

   b) We will have unbiased parameter estimates, but biased standard errors.

   c) The R^2 is a valid goodness-of-fit statistic, since we have accounted for heteroskedasticity when computing the standard errors.