Problem Set 8
Autocorrelation

Note: In those exercise based on estimations in which the data used is referred, the student should check in Gretl the presented results.

1. Consider the random variablesY_1, Y_2 and Y_3 such that

$$E(Y_1) = E(Y_2) = E(Y_3) = \mu$$
$$V(Y_1) = V(Y_2) = V(Y_3) = 2$$
$$C(Y_1, Y_2) = 1, C(Y_1, Y_3) = 0, C(Y_2, Y_3) = 0$$

Suppose we have one observation per each variable.

   a) Let m be the OLS estimator of μ. Calculate m and $V(m)$.
   b) Any linear and unbiased estimator of μ has the form

$$m^* = c_1Y_1 + c_2Y_2 + c_3Y_3,$$

   where $c_1 + c_2 + c_3 = 1$. Find the values of c_1, c_2, c_3 that minimice the variance, as well as the corresponding optimal estimator m^* and its variance. Does it equal the OLS esrtimator? Otherwise, interpret the result.
   c) Suppose that you use the conventional expression of the variance of the OLS estimator of μ (es decir, la media muestral), based on the variance of the sample mean

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (Y_i - m)^2$$

   with $n = 3$. Will s^2 be a biased estimator of $V(m)$? Will it be a consistent estimator? Justify.

2. Consider the random variables Y_1 and Y_2 such that

$$E(Y_1) = \mu,\ E(Y_2) = 2\mu$$
$$V(Y_1) = 3,\ V(Y_2) = 4$$
$$C(Y_1, Y_2) = 2$$

Suppose we have one observation per each variable.

   a) Obtain the minimum-variance linear unbiased estimator of μ.
   b) Obtain the OLS estimator of μ, and compare its mean and its variance with the ones of the earlier estimator.

3. The file GEW1 includes annual observations of two firms, General Electric (GE) and Westinghouse (WE) for the period 1935-1954. Consider the following variables: $V1$ = year (taking the values $1, \ldots, 20$), $V2$ = GE investment, $V3$ = lagged market value of GE, $V4$ = lagged capital stock of GE, $V5$ = WE investment, $V6$ = lagged market value of WE, $V7$ = lagged capital stock of WE. The variables $V2, \ldots, V7$ are measured in constant million dollars of 1947.
   We aim at constructing a simple model about firm's capital investment, relating its annual investment with its market value. Let Y = firm investment, and X = firm market value in the previous year.
   Assume
   $$E(Y|X) = \alpha + \beta X.$$
   a) For General Electric:
      (i) Estimate the simple regression model and comment the results.
      (ii) Generate a variable with the resiudals of the earlier regression e_t and the lagged residuals e_{t-1} . Test for serial error autocorrelation based on the OLS estimation of the linear projection
      $$e_t = \theta_0 + \theta_1 e_{t-1} + v_t.$$
      What can you conclude about the existence of error autocorrelation in the relationship between investment and market value? Do your conclusions coincide with the ones based on the Durbin-Watson test?
      (iii) Do your conclusions affect to the computation of standard errors reported in the estimation of (i)? In such a case, estimate the model using the appropriate standard errors.

   b) Repeat the analysis forWestinghouse.

4. When implementing the OLS linear regression with a sample of 100 quarterly observations of Y = sales, X_1 = advertising expenditure, and X_2 = R&D (research and development) expenditure, we get the following results:
   $$\begin{array}{llll} \widehat{Y} = & 1{,}70 & +1{,}25X_1 & +1{,}00X_2 \qquad R^2 = 0{,}73 \\ & (0{,}77) & (0{,}50) & (0{,}30) \end{array}$$
   $$\text{Durbin-Watson} \quad d = 0{,}80$$
   The researcher concluded that, at the 5 % significance level, advertising expenditures affected sales. Given the available information, do you agree such conclusion? Justify. (Hint: What is the approximate relation between the Durbin-Watson statistic and an estimate of the first-order error autocorrelation coefficient?).

5. The file TIM1 contains annual US data for the period 1959-1996 of the following variables: $V1$ = year, $V2$ = real GDP (in constant thousand million dollars of 1992), $V3$ = GDP price deflator (index = 100 in 1992), $V4$ = nominal money supply defined as *M1* (in thousand million dollars), $V5$ = real consumption (in constant thousand million dollars of 1992), $V6$ = 3-month Treasury-bills interest rate (in anual %), $V7$ = unemployment rate (in %), and

$V8$ = nominal money demand defined as *M2* (in thousand million dollars).
We will use *M1* (that includes coins and bank notes, and bamnk deposits) as definition of money amount, and take Y = log of real money demand = $\ln(100V4/V3)$, X_1 = log of real GDP = $ln(V2)$, X_2 = T-bill interest rate = $V6$. We assume that the classical regression model is valid for $E(Y|X_1, X_2) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$.

a) Suppose we can apply the classical regression model to $E(Y|X_1, X_2) = \beta_0+\beta_1 X_1+\beta_2 X_2$. Estimate the conditional mean function using OLS.

b) Your econometric package will report the Durbin-Watson statistic, d. Calculate $1-d/2$, the estimate of the first order error autocorrelation.

c) Take the model residuals, e_t, and regress them against its lagged value, e_{t-1} (include a constant). We will have 37 observations for resoduals. Call r the slope of this residual autorregression. Is the value of r close to $1-d/2$? Is the constant close to zero? (Justify). What is the result of the first-order autocorrealtion test?

d) Implement a test of no autocorrelation of both first and second order.

e) Repeat the model estimation using Newey-West autocorrelation-robust standard errors.