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Departamento de Matemáticas

Complex variable and transforms. Problems

Chapter 1: Complex variable

Section 1.4: Elementary functions

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1.4. ELEMENTARY FUNCTIONS

4.1. Write on exponential form the following complex numbers: 1, -1, i, $-i$, $1+i$, $1-i$, $-1+i$, $-1-i$.

4.2. Find the moduli and the principal values of the arguments (obtained by taking $\arg z \in (-\pi, \pi])$ of the following complex numbers: e^{2+i}, e^{2-3i}, e^{3+4i}, e^{-3-4i}, $-ae^{i\theta}$ $(a > 0,\ |\theta| \leq \pi)$, $e^{-i\theta}$ $(|\theta| \leq \pi)$, $e^{i\alpha} - e^{i\beta}$ $(0 \leq \beta < \alpha \leq 2\pi)$.

4.3. Find the values of: $\log(-e)$, 1^i, $\sin i$, $\cos i$, $\cos(2+i)$, $\tan(1+i)$, $\operatorname{cotan}(\pi/4 - i\log 2)$, $\operatorname{cotanh}(2+i)$, $\tanh(\log 3 + \pi i/4)$.

4.4. Let a be a complex number different from 0 and r a real number. Compute the set of values of a^r in the following cases:

a) r integer, b) $r = m/n$ rational, c) r irrational.

4.5. Compute the values of $(2^i)^2$, $(2^2)^i$ and 2^{2i}.

4.6. Find the values of the following powers: $1^{\sqrt{2}}$, $(-2)^{\sqrt{2}}$, 2^i, 1^{-i}, i^i, $(1-i)^{1+i}$, $(3-4i)^{1+i}$.

4.7. Prove that the following formulas hold for every complex numbers z, w:

a) $\sin(z+w) = \sin z \cos w + \cos z\ \sin w$.

b) $\tan(z+w) = \dfrac{\tan z + \tan w}{1 - \tan z \tan w}$.

c) $\cosh(z+w) = \cosh z \cosh w + \sinh z\ \sinh w$.

d) $\tanh(z+w) = \dfrac{\tanh z + \tanh w}{1 + \tanh z \tanh w}$.

e) $\sin(iz) = i\ \sinh z$, $\cos(iz) = \cosh z$, $\tan(iz) = i \tanh z$.

f) $\cos^2 z + \sin^2 z = 1$, $\cosh^2 z - \sinh^2 z = 1$.

4.8. Compute the inverses of the trigonometric and hyperbolic functions in terms of the logarithm. Compute also their derivatives.

4.9. Find every complex number satisfying each of the following equations:

a) $e^{4z} = i$, b) $\sin z = 3$, c) $\sin z = i$, d) $\operatorname{cotan} z = i + 1$, e) $\sin z = 0$, f) $\cos z = 0$, g) $\tan z = 0$, h) $e^{e^z} = 1$, i) $\sin z + \cos z = 2$, j) $\sin z - \cos z = i$.

4.10. a) Prove with an example that $\log(zw) \neq \log z + \log w$, where log denotes the principal value of the logarithm (obtained by taking $\arg z \in (-\pi, \pi]$).

b) Prove that the formula $\log(zw) = \log z + \log w$ holds if $\log z$ denotes the set of every logarithm of z.