COMPLEX VARIABLE AND TRANSFORMS	TEST 2

Problem 1 (1) Compute the Fourier series of the function $f(x) = x$ defined on $[-L, L]$.
(2) At which points x of the interval $[-L, L]$ does the Fourier series of f converge to $f(x)$?.
(3) Compute the Fourier sine series of the function $f(x) = x$ defined on $[0, L]$.

Problem 2 Find a solution of the initial value problem for the diffusion equation with absorption:

$$\begin{cases} \frac{\partial}{\partial t} u(x, t) = k \frac{\partial^2}{\partial x^2} u(x, t) - c\, u(x, t), & \text{if } x \in \mathbb{R}\,, \ t > 0\,, \\ u(x, 0) = f(x)\,, & \text{if } x \in \mathbb{R}\,. \end{cases}$$

Problem 3 Find a function satisfying $f(0) = 1$ and

$$f'(x) = 1 - \int_0^x f(x - s)\, e^{-2s}\, ds\,,$$

by using the Laplace transform.

Problem 4 Solve the following difference equation by using the Z-transform,

$$f_{n+2} - f_{n+1} - 2f_n = 0, \qquad \text{with} \qquad f_0 = 0,\ f_1 = 1.$$

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