uc3m | Universidad **Carlos III** de Madrid
Departamento de **Matemáticas**

COMPLEX VARIABLE AND TRANSFORMS — TEST 3

Problem 1 Find the radius of convergence of the series (without using Stirling formula):

$$(1) \quad \sum_{n=1}^{\infty} n z^{n^3} = z + 2z^8 + 3z^{27} + \dots, \qquad (2) \quad \sum_{n=1}^{\infty} \frac{(2n)!}{n!(n+3)!} z^n .$$

Problem 2 (1) Obtain the Laurent's series on the annulus $\{1 < |z| < 2\}$ of the function

$$f(z) = \frac{2z-1}{z^2 - z - 2} .$$

(2) Obtain the Laurent's series of the same function f on the annulus $\{2 < |z| < \infty\} = \{2 < |z|\}$.

Problem 3 (1) Compute the principal value of the following integral for $\omega > 0$, checking that the hypotheses (of the theorem that you use) hold:

$$p.v. \int_{-\infty}^{\infty} \frac{e^{i\omega x}}{x(x^2+4)} \, dx .$$

(2) Compute this principal value for $\omega < 0$ by using the previous item.

Problem 4 Find a solution of the initial value problem for the heat equation:

$$\begin{cases} \frac{\partial}{\partial t} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t) , & \text{if } x \in \mathbb{R} , \ t > 0 , \\ u(x,0) = e^{-x^2} , & \text{if } x \in \mathbb{R} . \end{cases}$$

Problem 5 Solve the following equation

$$f(t) = \cos t + \int_0^t e^{-s} f(t-s) ds$$

by using the Laplace transform.

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