

**Problem 1** Find the radius of convergence of the series:

$$\sum_{n=0}^{\infty} z^{2^n} = z + z^2 + z^4 + z^8 + \dots$$

**Problem 2** Find the points in which the following series is convergent:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\log n} z^{2n}.$$

*Hint:* Consider a change of variable.

**Problem 3** Find the power series of the following function about 0:

$$\int_0^z \frac{\sin w}{w} dw.$$

*Hint:* Recall that

$$\sin z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}.$$

**Problem 4** Compute the integral

$$\int_{\gamma} \frac{z}{1 - \cos z} dz,$$

where  $\gamma$  is the circumference centered at 0 with radius 5, positively oriented.

**Problem 5** Compute the following integral, checking that the hypotheses (of the theorem that you use) hold:

$$\int_{-\infty}^{\infty} \frac{\cos(ax)}{x^2 + b^2} dx, \quad a, b > 0.$$

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