uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

CALCULUS I - OpenCourseWare

All Units Final Exam.

Problem 1. [1.5 points] Prove that the following sequence is convergent and calculate its limit.

$$\begin{cases} a_1 = 1 \\ a_{n+1} = \dfrac{1}{3 - a_n} \end{cases}$$

Problem 2. [2.5 points] Study the convergence of the following series of real numbers:

a) (1 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{a^n\, n!} \; , \; a < e$ **b) (1.5 pts)** $\displaystyle\sum_{n=1}^{\infty} \left(n \sin\frac{1}{n}\right)^{n^2}$

Problem 3. [3.5 points] Consider the function:

$$f(x) = \begin{cases} \dfrac{ax^2 + x + 1 - e^{bx}}{bx^2 + ax + a} & x \neq 0 \\ -1/2 & x = 0 \end{cases} \quad ; \quad a, b \in \mathbb{R}$$

a) (0.75 pts) Find the values of a and b that make f continuous in $\mathbb{R}$.

b) (0.75 pts) Calculate, if possible, the value of $f'(0)$ for the resulting function in part a).

c) (1 pts) Taking $a = 2$, $b = 0$ calculate, if possible, the Taylor polynomial of second degree for $f(x)$ centered at $x = -2$.

d) (1 pts) Give an upper bound for the error made when using the polynomial obtained in part c) to approximate $f(-3)$.

Problem 4. [2.5 points] Calculate the following integrals:

a) (1 pts) $\displaystyle\int_1^{64} \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$ **b) (1.5 pts)** $\displaystyle\int \frac{-2x + 4}{(x^2+1)(x-1)^2}\, dx$
