



Problem 1. [1.5 points] Show that the following sequence is convergent and calculate its limit.

$$\begin{cases} a_1 = 1 \\ a_{n+1} = \sqrt{2a_n + 3} \end{cases}$$

Problem 2. [2 points] Calculate the following limits, where $n \in \mathbb{N}$ and $x \in \mathbb{R}$.

a) (1 pt) $\lim_{n \rightarrow \infty} \frac{\frac{2}{1} + \frac{3^2}{2} + \frac{4^3}{3^2} + \cdots + \frac{(n+1)^n}{n^{n-1}}}{n^2}$ b) (1 pt) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{x \sin x} \right)$

Problem 3. [2.5 points] Let f be the following function:

$$f(x) = \begin{cases} \frac{a}{b \sin^2(x-1) + 1} & x \in [0, 1) \\ \arctan(x) - \log\left(\frac{x}{x-b}\right) & x \geq 1 \end{cases} ; \quad a, b \in \mathbb{R}$$

- a) (1 pt) Find (with proper justification) the relationship between a and b so that $f(x)$ is continuous in $x = 1$.
- b) (1 pt) Calculate, if possible, the values of a and b so that the resulting function in a) is differentiable in $x = 1$.
- c) (0.50 pts) Graph schematically the function obtained in b). (It is NOT necessary to find the exact values of any characteristic points of the function).

Problem 4. [2 points] Determine the area between the curves $y = x - 1$ and $x = 3 - y^2$.

Problem 5. [2 points] Calculate the following integrals:

a) (1 pt) $\int e^{2x} \sin x \, dx$ b) (1 pt) $\int \frac{3x + 5}{x^3 - x^2 - x + 1} \, dx$
