uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

Problem 1. **[1.5 points]** Show that the following sequence is convergent and find its limit.

$$a_1 = 1, \quad a_{n+1} = \log(1 + a_n)$$

HINT: Take into account the graphs of the functions $1 + x$ and e^x.

Problem 2. **[1.5 points]** Study the convergence of the following series:

a) (0.75 pts) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$ **b) (0.75 pts)** $\displaystyle\sum_{n=1}^{\infty} \sin(1+n)$

Problem 3. **[1.5 points]** Calculate the following limits:

a) (0.75 pts) $\displaystyle\lim_{x \to 0} \frac{\sin^2 x \arctan x}{(\log(x+1))^2 \left(\sqrt[3]{x+1} - 1 \right)}$ **b) (0.75 pts)** $\displaystyle\lim_{x \to 0} \frac{2e^{x^2} + \cos x - 3}{x^2}$

Problem 4. **[1.5 points]** Study the relative extrema of the following function in the interval $[1, \infty)$.

$$f(x) = \int_0^{x-1} \left(e^{-t^2} - e^{-2t} \right) dt$$

Problem 5. **[2 points]** Calculate the following integrals

a) (1 pts) $\displaystyle\int \frac{x^2}{x^2 - 4}\, dx$ **b) (1 pts)** $\displaystyle\int \frac{\log(\log x)}{x \log x}\, dx$

HINT: in b) use the change of variable $t = \log x$ repeatedly.

Problem 6. **[2 points]** Calculate the length of the curve given by

$$\begin{cases} x(t) = 2 + \dfrac{t^3}{3} \\ y(t) = \dfrac{t^2}{2} - 1 \end{cases}$$

with $0 \leq t \leq 1$.
