uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

CALCULUS I - OpenCourseWare

All Units Final Exam.

Problem 1. [**1 point**] Find the volume of the solid formed by revolving the region between the curves $\sqrt{x}$ and x around the y axis.

Problem 2. [**1.5 points**] Given the function $f(x) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x e^{-t^2}\, dt$,

a) [**0.5 points**] Show that it is increasing in $(-\infty, \infty)$.

b) [**1 point**] Show that $y = e^{x^2}(1 + \sqrt{\pi} f(x))$ satisfies the differential equation $\dfrac{dy}{dx} - 2xy = 2$ and that $y(0) = 1$.

Problem 3. [**1 point**] Find $\displaystyle\lim_{x\to 0} \frac{xf(x)}{(f(x)-1)(x^2+f(x)^2)}$, considering that $\displaystyle\lim_{x\to 0} f(x) = 0$ and that f is twice differentiable.

HINT: Express the result in terms of $f'(0)$.

Problem 4. [**2 points**] Sum the following series: a) [**1 point**] $\displaystyle\sum_{n=1}^{\infty} \frac{n2^{n-3} + 5^n}{7^{n-1}}$

b) [**1 point**] $\displaystyle\sum_{n=2}^{\infty} (e^{1/n} - e^{1/(n-1)})$

Problem 5. [**2 points**] Calculate the following integrals:

a) [**1 point**] $\displaystyle\int_0^{\pi} x^2 \cos x\, dx$ b) [**1 point**] $\displaystyle\int \frac{4x}{(x^2+1)(x^2+2x+3)}\, dx$

Problem 6. [**2.5 points**] Given the recurrence:

$$a_{n+1} = 2 + \frac{3}{a_n}, \quad a_1 = 5/2,$$

a) [**0.5 points**] Show that $a_n > 2$ for all $n \in \mathbb{N}$.

b) [**0.5 points**] Supposing $a = \lim_{n\to\infty} a_n$ exists, find it.

c) [**1 point**] Show that the sequence is alternating, i.e $(a_{n+1} - a_n)(a_n - a_{n-1}) < 0$.

d) [**0.5 points**] Show that $|a_{n+1} - a| < \dfrac{1}{2}|a_n - a|$ and prove that the sequence is convergent.
