



Problem 1. [2.5 points] Find the inverse of $f(x) = x^2 - x + 1$, assuming $x \geq 1/2$. What is the domain of $f^{-1}(x)$?

Problem 2. [2.5 points] Using the induction principle, prove:

$$(n+1)^2 + (n+2)^2 + (n+3)^3 + \dots + (2n)^2 = \frac{n(2n+1)(7n+1)}{6}$$

Problem 3. [2.5 points] Consider the following sequence:

$$\begin{cases} a_0 &= 3 \\ a_{n+1} &= \frac{1}{2} \left(a_n + \frac{2}{a_n} \right) \end{cases}$$

- a) Suppose the sequence has a limit ℓ and compute its possible values.
 - b) Show that the sequence is bounded below by ℓ (HINT: show that $a_n - \ell \geq 0$).
 - c) Show that the sequence is monotonically decreasing.
 - d) Is the sequence convergent? Why? What is its limit?
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Problem 4. [2.5 points] Sum the following series:

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 4n}$$
