



Problem 1. [2 points] Prove, using induction, that $n^3 + 5n$ is a multiple of 6 for every $n \in \mathbb{N}$.

(HINT: $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$).

Problem 2. [2 points] What is the domain of the function $f(x) = \arcsin(\exp(x^2 - x - 20))$? Is f surjective? And injective? If so, find its inverse.

Problem 3. [3 points] Study the convergence of the following sequence, and find its limit if it exists:

$$a_{n+1} = \sqrt{6 + a_n}, \quad a_1 = 4.$$

Problem 4. [3 points] Study the convergence of the following series of real numbers:

a) [1.5 points]

$$\sum_{n=1}^{\infty} \frac{n^3 2^{n+3}}{7^{n-1}}$$

b) [1.5 points]

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$
