uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

Problem 1. **[2 points]** Prove, using induction, that $4^{2n+1} + 3^{n+2}$ is a multiple of 13 for every $n \in \mathbb{N}$.

Problem 2. **[2 points]** What is the domain of the function $f(x) = \log(\sinh(x^2 - x - 20))$? Is f surjective? And injective? If so, find its inverse.

Problem 3. **[3 points]** Study the convergence of the following sequence, and find its limit if it exists:

$$a_{n+1} = \frac{a_n^2}{1 + a_n}, \quad a_1 = 1.$$

Problem 4. **[3 points]** Study the convergence of the following series of real numbers:

a) [1.5 points]

$$\sum_{n=1}^{\infty} \frac{(2n)!}{n!(2n)^n}$$

b) [1.5 points]

$$\sum_{n=1}^{\infty} \left(\frac{ne}{n+1} \right)^n$$
