



Problem 1. [2 points] Find the following limits:

a) [1 point] $\lim_{x \rightarrow 0} \frac{e^{2x} - \sin(2x) - 1}{x \tan x}$. b) [1 point] $\lim_{x \rightarrow 0} \frac{x + \sin(\pi x)}{x - \sin(\pi x)}$.

Problem 2. [3 points] Given the function $f(x) = \begin{cases} \frac{x-1}{x^3} + \beta & \text{if } x > 1, \\ \arctan(\log x) & \text{if } x \leq 1. \end{cases}$

- a) [0.5 points] Find the domain of f .
- b) [1.5 points] Find β so that f is differentiable in its domain.
- c) [1 point] Taking $\beta = -1$, find the extrema of f .

Problem 3. [3 points] Given the function $f(x) = \log(\sqrt{1+x})$

- a) [1 point] Find its Taylor polynomial of degree 3 at $x = 0$.
- b) [1 point] Use that polynomial to approximate the value of $\log(\sqrt{1.1})$. Show that the error of the approximation is less than 10^{-4} .
- c) [1 point] Find $f'(x)$ and compute its Taylor polynomial of degree 2 at $x = 0$.

Problem 4. [2 points] Given the series $S = \sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)!}$

- a) [1 point] Show that it is absolutely convergent.
- b) [1 point] Compute its value (HINT: find a function whose Taylor series is similar to S).