



Problem 1. [4 points] Consider the function

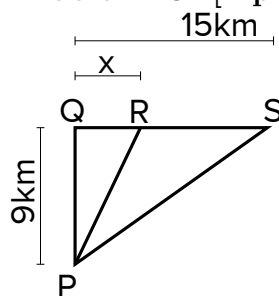
$$f(x) = \begin{cases} \log\left(\frac{1}{|x|}\right), & \text{if } |x| > 1, \\ a \cos(b\pi x), & \text{if } |x| \leq 1 \end{cases}$$

- Find (if possible) the values of a and b that make f continuous and differentiable everywhere. [1 point]
- Using $a = 1, b = 1$, find the relative maxima and minima of f . [1 point]
- Using $a = 1, b = 1$, find the sets where f is convex. [1 point]
- Draw a rough sketch of the function using the information obtained above (and either the values of a and b obtained in part a) or those in part b). [1 point]

Problem 2. [4 points]

- Calculate the integral $\int_0^1 \cos^2 x \, dx$. [1 point]
- Find the Taylor polynomial of degree 5 for $\cos^2 x$, centered at $x = 0$. [1 point]
- Use the polynomial in part b) to approximate the integral in part a). [1 point]
- What is the error made by this approximation? (HINT: find the error $R(x)$ of your Taylor approximation using the formula for the remainder. Then integrate $\int_0^1 R(x) \, dx$). [1 point]

Problem 3. [2 points]



A woman is on an island at point P and wants to reach point S in the shore. In order to reach the shore she will use a boat, rowing at a speed of 3 km/h. She will land at point R, and from R to S she will walk at a speed of 5 km/h. Where should she land her boat so that she takes the least time to reach S?