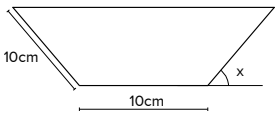
uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

Problem 1. [2 points]



A pipe with a trapezoid section has the dimensions shown in the figure. Determine the value of the angle x (between 0 and π) so that the volume that the pipe carries is maximal. (HINT: $\cos(\pi \pm x) = -\cos x$)

Problem 2. [3 points] Consider the function $f(x) = \arctan\left(\dfrac{1}{1-x}\right)$

a) Find the sets where it is continuous and differentiable. Find its relative maxima and minima, and the sets where it is convex. **[1.5 points]**

b) Draw a rough sketch of the function using the information obtained above. **[0.5 points]**

c) Find the third degree Taylor polynomial of the function centered at $x = 0$. **[1 point]**

Problem 3. [3 points]

a) Approximate the value of $\dfrac{1}{\sqrt[3]{9}}$ using the second degree Taylor expansion of an appropriate function. **[1.5 points]**

b) What is the (approximate) maximum value of the error made in this approximation? **[1 point]**

c) What should be the degree of the approximation in order for the error to be smaller than 10^{-4}? **[0.5 points]**

Problem 4. [2 points] Calculate the following limits:

a) [1 point]

$$\lim_{x\to 0} \frac{\sin^2 x - (e^x - 1)^2}{\log(1+x^3)}$$

b) [1 point]

$$\lim_{x\to\infty} x^2 \left(\sqrt{1-1/x} - x\log(1+1/x)\right)$$

(HINT: $\sqrt{1-z} = 1 - z/2 - z^2/8 + o(z^2)(z \to 0)$)
