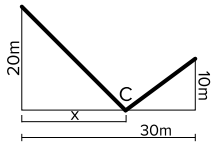
uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

Problem 1. [2 points]



A cable is bound to two posts 30 meters apart, and it is held fixed to the floor at a point C between them (see figure). The height of the leftmost post is 20 meters, while the height of the rightmost one is 10 meters. What is the distance x from the leftmost post at which we have to bind the cable so that its length is minimal?

Problem 2. [3 points] Consider the function $f(x) = \begin{cases} |x|, & \text{if } |x| < 1, \\ \frac{1}{1+(x+1)^2}, & \text{if } |x| \geq 1 \end{cases}$

a) Find the sets where it is continuous and differentiable. Find its relative maxima and minima, and the sets where it is convex. **[1.5 points]**

b) Draw a rough sketch of the function using the information obtained above. **[0.5 points]**

c) Find the second degree Taylor polynomial of the function centered at $x = 2$. **[1 point]**

Problem 3. [3 points]

a) Approximate the value of $\log 0.9$ using the third degree Taylor expansion of an appropriate function. **[1.5 points]**

b) What is the (approximate) maximum value of the error made in this approximation? **[1 point]**

c) What should be the degree of the approximation in order for the error to be smaller than 10^{-5}? **[0.5 points]**

Problem 4. [2 points] Calculate the following limits:

a) [1 point]

$$\lim_{x\to 0} \frac{\sin x \cos x - \tan x}{x^2 \log(1+x)}$$

(HINT: $\tan x = x + x^3/3 + o(x^3)$).

b) [1 point]

$$\lim_{x\to\infty} \frac{\cos(\frac{1}{x}) - 1 - \frac{1}{2x^2}}{\sin(\frac{1}{6x^2})}$$
