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Calculus I

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Unit 5. Limit of a Function

Exercises



Problems

Problem 5.1 Calculate the following limits, simplifying the common factors that numerator and denominator may contain:

$$\begin{aligned} \text{(i)} \quad & \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}, \quad n \in \mathbb{N}; & \text{(iii)} \quad & \lim_{x \rightarrow 64} \frac{\sqrt{x} - 8}{\sqrt[3]{x} - 4}; & \text{(v)} \quad & \lim_{x \rightarrow 0} \frac{\frac{1}{(1-x)^3} - 1}{x}; \\ \text{(ii)} \quad & \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x - a}; & \text{(iv)} \quad & \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x^2}}{x^2}; & \text{(vi)} \quad & \lim_{x \rightarrow 1} \left(\frac{1}{\sqrt{x} - 1} - \frac{2}{x - 1} \right). \end{aligned}$$

Problem 5.2 Calculate the following limits:

$$\begin{aligned} \text{(i)} \quad & \lim_{x \rightarrow 0} \frac{(\sin 2x^3)^2}{x^6}; & \text{(v)} \quad & \lim_{x \rightarrow 0} \frac{\log(1 - 2x)}{\sin x}; & \text{(ix)} \quad & \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \right)^{\frac{\sin x}{\sin x - x}}; \\ \text{(ii)} \quad & \lim_{x \rightarrow 0} \frac{\tan x^2 + 2x}{x + x^2}; & \text{(vi)} \quad & \lim_{x \rightarrow 0} (1 + \sin x)^{2/x}; & \text{(x)} \quad & \lim_{x \rightarrow 0} (\cos x)^{1/x^2}; \\ \text{(iii)} \quad & \lim_{x \rightarrow 0} \frac{\sin(x+a) - \sin a}{x}; & \text{(vii)} \quad & \lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{x - \sin x}; & \text{(xi)} \quad & \lim_{x \rightarrow \pi} \frac{1 - \sin(x/2)}{(x - \pi)^2}; \\ \text{(iv)} \quad & \lim_{x \rightarrow 0} (1+x)^{1/x}; & \text{(viii)} \quad & \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}; & \text{(xii)} \quad & \lim_{x \rightarrow 0} \frac{a^x - b^x}{x}. \end{aligned}$$

Problem 5.3 Calculate the following limits:

$$\begin{aligned} \text{(i)} \quad & \lim_{x \rightarrow \infty} \frac{x^3 + 4x - 7}{7x^2 - \sqrt{2x^6 + x^5}}; & \text{(iv)} \quad & \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 4x} - x \right); & \text{(vii)} \quad & \lim_{x \rightarrow \pm\infty} \tanh x; \\ \text{(ii)} \quad & \lim_{x \rightarrow \infty} \frac{x + \sin x^3}{5x + 6}; & \text{(v)} \quad & \lim_{x \rightarrow \pm\infty} \frac{e^x}{e^x - 1}; & \text{(viii)} \quad & \lim_{x \rightarrow \pm\infty} \frac{e^x}{\sinh x}; \\ \text{(iii)} \quad & \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}}; & \text{(vi)} \quad & \lim_{x \rightarrow \pm\infty} \frac{x - 2}{\sqrt{4x^2 + 1}}; & \text{(ix)} \quad & \lim_{x \rightarrow \pm\infty} \left(\frac{2x + 7}{2x - 6} \right)^{\sqrt{4x^2 + x - 3}}. \end{aligned}$$

Problem 5.4 Calculate the one-sided limits:

$$\begin{aligned} \text{(i)} \quad & \lim_{x \rightarrow 0^\pm} \left(\frac{1}{x} \right)^{\lfloor x \rfloor}; & \text{(ii)} \quad & \lim_{x \rightarrow 0^\pm} e^{1/x}; & \text{(iii)} \quad & \lim_{x \rightarrow 0^\pm} \frac{1 - e^{1/x}}{1 + e^{1/x}}. \end{aligned}$$