

OpenCourseWare

CALCULUS – Sets of numbers

Filippo Terragni, Eduardo Sánchez Villaseñor, Manuel Carretero Cerrajero

Problem 1.1. Find all values of $x \in \mathbb{R}$ satisfying the following conditions.

- 1) $x^2 + \frac{2}{x} > 3$.
- 2) $|\sqrt{x} - 2| \leq 3$.
- 3) $-8 \leq |x - 5| - |x - 3| \leq 8$.
- 4) $|x - 3| \leq 8$.
- 5) $0 < |x - 2| < \frac{1}{2}$.
- 6) $x^2 - 5x + 6 \geq 0$.
- 7) $x^3(x + 3)(x - 5) > 0$.
- 8) $\frac{2x + 8}{x^2 + 8x + 7} > 0$.
- 9) $|x - 1| + |x - 2| > 1$.
- 10) $|x - 1||x + 2| = 3$.

Problem 1.2. Find the supremum, infimum, maximum, and minimum (when they exist) of the following subsets of \mathbb{R} .

- 1) $A_1 = \{1/n : n \in \mathbb{N}\}$.
- 2) $A_2 = \{1/n : n \in \mathbb{Z}, n \neq 0\}$.
- 3) $A_3 = \{x \in \mathbb{Q} : 0 \leq x \leq \sqrt{2}\}$.
- 4) $A_4 = \{x \in \mathbb{R} : x^2 + x + 1 \geq 0\}$.
- 5) $A_5 = \{x \in \mathbb{R} : x^2 + x - 1 < 0\}$.
- 6) $A_6 = \{x \in \mathbb{R} : x < 0, x^2 + x - 1 < 0\}$.
- 7) $A_7 = \{1/n + (-1)^n : n \in \mathbb{N}\}$.
- 8) $A_8 = \{x \in \mathbb{R} : 3x^2 - 10x + 3 < 0\}$.
- 9) $A_9 = \{x \in \mathbb{R} : (x - a)(x - b)(x - c)(x - d) < 0; a, b, c, d \in \mathbb{R}, a < b < c < d\}$.
- 10) $A_{10} = \{2^{-p} + 5^{-q} : p, q \in \mathbb{N}\}$.

Problem 1.3. Prove the following properties by using the most suitable method.

1) $\sqrt{2}$ is an irrational number .

$$2) \sum_{n=0}^N r^n = \frac{1-r^{N+1}}{1-r}, \quad r \in \mathbb{R}, \quad r \neq 1, \quad N \in \mathbb{N}.$$

$$3) \sum_{n=1}^N n = \frac{N(N+1)}{2}, \quad N \in \mathbb{N}.$$

$$4) 0 < x < y \implies x < \sqrt{xy} < \frac{x+y}{2} < y \quad (x, y \in \mathbb{R}).$$

$$5) 0 < x < y \implies \frac{x}{y} < \frac{x+k}{y+k}, \quad \forall k > 0 \quad (x, y \in \mathbb{R}).$$

$$6) |x+y| = |x|+|y| \iff xy \geq 0 \quad (x, y \in \mathbb{R}).$$