uc3m | Universidad Carlos III de Madrid

OpenCourseWare

CALCULUS – Real functions: limits and continuity

Filippo Terragni, Eduardo Sánchez Villaseñor, Manuel Carretero Cerrajero

Problem 3.1. Sketch the graph of the following functions.

$$\begin{aligned} 1)\quad f(x) &= \sqrt{x-3}\,. \\ 2)\quad f(x) &= x^2-4x+5\,. \\ 3)\quad f(x) &= \left|\,(x-2)^3+1\,\right|\,. \end{aligned}$$

Problem 3.2. Find the domain and image of the following functions. Then, in each case, study the continuity of $f(x)$ and sketch its graph.

$$\begin{aligned} 1)\quad f(x) &= \lfloor x \rfloor\,. \\ 2)\quad f(x) &= x - \lfloor x \rfloor\,. \\ 3)\quad f(x) &= \sqrt{x-\lfloor x \rfloor}\,. \\ 4)\quad f(x) &= \lfloor x \rfloor + \sqrt{x-\lfloor x \rfloor}\,. \\ 5)\quad f(x) &= \left\lfloor \frac{1}{x} \right\rfloor\,. \end{aligned}$$

Problem 3.3. Study the continuity of the following functions.

$$\begin{aligned} 1)\quad f(x) &= \frac{e^x+2\cos(x)-8x+5}{e^x+\sin^2(x)+5}\,. \\ 2)\quad f(x) &= \sqrt{x^4+3}+e^{-x^2+\cos(x)}\sin(4x^5+3x^2+2x-5+\cos(x))+2\arctan(3^x-5)\,. \\ 3)\quad f(x) &= e^{4/x}+x^4-7\,. \\ 4)\quad f(x) &= \arccos^5(x)\,. \\ 5)\quad f(x) &= (x-3)\ln(9x-4)\,. \\ 6)\quad f(x) &= (4x^6+3x^3-2x+6)\ln(x)+\ln(9x-4)\arccos(x)\,. \end{aligned}$$

Problem 3.4. Analyze the continuity of the given functions.

$$f(x) = \begin{cases} x\cos\left(\dfrac{1}{x}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

$$g(x) = \begin{cases} e^x, & \text{if } x \leq 0, \\ x^2 - x, & \text{if } 0 < x < 1, \\ \cos\left(\pi|2 - x^2|\right) + 1, & \text{if } x \geq 1. \end{cases}$$

$$h(x) = \begin{cases} x^3 - x + 5, & \text{if } x < 0, \\ e^x + \sin(x), & \text{if } x \geq 0. \end{cases}$$

Problem 3.5. Prove that the following function is bounded.

$$f(x) = \begin{cases} e^{1/x}, & \text{if } -7 \leq x < 0, \\ 0, & \text{if } 0 \leq x \leq 5. \end{cases}$$

Problem 3.6. Prove that the equation

$$\cos(x) = x$$

has *some* real solution.

Problem 3.7. Prove the following theorems.

1) Let $f : [0, 1] \to [0, 1]$ be a continuous function. Then, there is $x_0 \in (0, 1)$ such that $f(x_0) = x_0$.

2) Let f, $g : [x_1, x_2] \to \mathbb{R}$ be two continuous functions such that $f(x_1) > g(x_1)$ and $f(x_2) < g(x_2)$. Then, there is $x_0 \in (x_1, x_2)$ such that $f(x_0) = g(x_0)$.