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CALCULUS – Local & global behavior of a real function

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Problem 7.1. Find and classify the *local* extrema of the following functions.

- $f(x) = 2x^3 - 3x^2 - 12x + 5$.
- $f(x) = \dfrac{x+3}{x-2}$.
- $f(x) = x^2 e^{-2x}$.

Problem 7.2. Consider the function $f(x) = |x^3(x-4)| - 1$.

- Study where $f(x)$ is (strictly) increasing.
- Find and classify the *local* extrema of $f(x)$.
- Prove that the equation $f(x) = 0$ has a unique (real) solution in the interval $(0, 1)$.

Problem 7.3. What is the *local* behavior of the function $f(x) = x^{101} + x^{51} + x + 1$ in a neighborhood of $x = 0$?

Problem 7.4. Study the concavity of the given functions.

$$f(x) = (x-2)\, x^{2/3}.$$
$$f(x) = x\,(x-2)^{3/2}.$$
$$f(x) = |x|\, e^{|x|}.$$
$$f(x) = \ln(x^2 - 6x + 8).$$

Problem 7.5. Study the *local* behavior in a neighborhood of $x = 0$ of the function

$$f(x) = x^4 \sqrt{1+x^2}\, (\cos(2x) - 1)^2 \,.$$

Problem 7.6. Let

$$f(x) = \begin{cases} \alpha + x + x^2 \,, & \text{if} \quad x < 0 \,, \\ \beta \sin(x) \,, & \text{if} \quad x \geq 0 \,, \end{cases}$$

where $\alpha\,,\, \beta \in \mathbb{R}$.

- For $x < 0$, find the intervals where $f(x)$ is decreasing.
- Find the values of α and β such that $f(x)$ is differentiable at $x = 0$.
- Let $\alpha = -1$ and $\beta = 1$. Find and classify the *global* extrema of $f(x)$.

Problem 7.7. Let $f(x) = 3x^4 - 4x^3 + 1$.

- Find and classify the critical points of $f(x)$.
- Determine the intervals where $f(x)$ is increasing.
- Calculate the inflection points of $f(x)$.
- Study the concavity of $f(x)$.

Problem 7.8. Find the *global* extrema of the following functions in the indicated intervals.

$$f(x) = \left| \frac{x}{\sqrt{2}} \right| + \cos(x) \,, \quad \text{with } x \in [-\pi, \pi] \,.$$

$$f(x) = 2x^{5/3} + 5x^{2/3} \,, \quad \text{with } x \in [-2, 1] \,.$$

Problem 7.9. Sketch the graph of the functions $f(x) = e^x \sin(x)$ and $g(x) = x^2\, e^x$.

Problem 7.10. Sketch the graph of the function

$$f(x) = x + \ln\left(|x^2 - 1|\right) .$$

Then, without any additional calculation, sketch the graph of the function

$$g(x) = \left| x + \ln\left(|x^2 - 1|\right) \right| .$$