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CALCULUS – Improper integrals

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Problem 9.1. Study the convergence of the following improper integrals.

$$\int_3^{+\infty} \frac{\ln^2(x)}{x}\,dx\,.$$
$$\int_1^{+\infty} \sin\left(\frac{1}{x}\right)\,dx\,.$$
$$\int_1^{+\infty} \frac{\sin(x)}{1+x^3}\,dx\,.$$
$$\int_1^{+\infty} \frac{dx}{x^\alpha\sqrt{1+x^2}}\,,\quad \alpha>0\,.$$
$$\int_1^{+\infty} \left(\frac{1}{\sqrt{x}} - \arctan\left(\frac{1}{\sqrt{x}}\right)\right)\,dx\,.$$
$$\int_2^{7} \frac{dx}{x^3-8}\,.$$
$$\int_0^{2} \frac{\arctan(x)}{x+x^2}\,dx\,.$$
$$\int_1^{2} \frac{\ln(x)+x-1}{(x-1)^{3/2}}\,dx\,.$$
$$\int_1^{+\infty} \frac{x}{\sqrt{x^4-1}}\,dx\,.$$
$$\int_0^{+\infty} \frac{1-\cos(x)}{x^3\,\ln(x)}\,dx\,.$$

Problem 9.2. Study the convergence of the given improper integrals (*exponential probability distribution*).

$$\int_0^{+\infty} e^{-x}\,dx\,.$$

$$\int_0^{+\infty} x^n e^{-x}\, dx\,, \quad n \in \mathbb{N}\,.$$
$$\int_0^{+\infty} \lambda e^{-\lambda x}\, dx\,, \quad \lambda > 0\,.$$
$$\int_0^{+\infty} \lambda x^n e^{-\lambda x}\, dx\,, \quad \lambda > 0\,, \quad n \in \mathbb{N}\,.$$

Problem 9.3. Analyze the convergence of the indicated improper integrals (*normal probability distribution*).

$$\int_{-\infty}^{+\infty} e^{-x^2}\, dx\,.$$
$$\int_{-\infty}^{+\infty} x^n e^{-x^2}\, dx\,, \quad n \in \mathbb{N}\,.$$
$$\int_{-\infty}^{+\infty} e^{-\frac{(x-3)^2}{4}}\, dx\,.$$
$$\int_{-\infty}^{+\infty} (x-3)^n e^{-\frac{(x-3)^2}{4}}\, dx\,, \quad n \in \mathbb{N}\,.$$
$$\int_{-\infty}^{+\infty} \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\, dx\,, \quad \mu \in \mathbb{R}\,, \quad \sigma > 0\,.$$
$$\int_{-\infty}^{+\infty} x^n \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\, dx\,, \quad \mu \in \mathbb{R}\,, \quad \sigma > 0\,, \quad n \in \mathbb{N}\,.$$

Problem 9.4. Study the convergence of the given improper integrals (*log-normal probability distribution*).

$$\frac{1}{\sigma\sqrt{2\pi}} \int_0^{+\infty} \frac{e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}}}{x}\, dx\,, \quad \mu \in \mathbb{R}\,, \quad \sigma > 0\,.$$
$$\frac{1}{\sigma\sqrt{2\pi}} \int_0^{+\infty} x^n e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}}\, dx\,, \quad \mu \in \mathbb{R}\,, \quad \sigma > 0\,, \quad n \in \mathbb{N}\,.$$

Problem 9.5. Study whether the following improper integrals converge or not (*beta probability distribution*).

$$\int_0^1 \frac{dx}{\sqrt{x}\sqrt{1-x}}\,.$$
$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\, dx\,, \quad \alpha,\, \beta > 0\,.$$
$$\int_0^1 x^n\, x^{\alpha-1}(1-x)^{\beta-1}\, dx\,, \quad \alpha,\, \beta > 0\,, \quad n \in \mathbb{N}\,.$$

Problem 9.6. Study the convergence of the given improper integral (*F probability distribution*).

$$\int_0^{+\infty} x^{\frac{n_1}{2}-1}\left(1+\frac{n_1}{n_2}x\right)^{-\frac{n_1+n_2}{2}} dx\,, \quad n_1,\, n_2 \in \mathbb{N}\,.$$