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CALCULUS – EVALUATION TEST 10

Filippo Terragni, Eduardo Sánchez Villaseñor, Manuel Carretero Cerrajero

Problem 1. Consider the *recursive* sequence $(a_n)_{n\in\mathbb{N}}$ defined as

$$a_1 = 1\,; \quad a_{n+1} = 3 - \frac{1}{a_n}\,, \text{ with } n = 1, 2, \dots .$$

(a) Prove that the sequence is increasing and bounded above by 3.

(b) Calculate $\lim_{n\to\infty} a_n$.

Problem 2. Analyze the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3\,\ln(n^2)}{(n+1)!}$.

Problem 3. Let $f(x) = \sin(x)$.

(a) Use the Taylor polynomial of degree 2 about $a = \pi/2$ for $f(x)$ to approximate $\sin(\pi/2 + 0.1)$ and find an *upper bound* for the involved error.

(b) Consider the Taylor polynomial of degree $n \in \mathbb{N}$ about $a = \pi/2$ for $f(x)$ and apply the change of variable $s = x - \pi/2$. Then, do you recognize the resulting Taylor polynomial?

Problem 4. Calculate the exact number of real solutions of the equation

$$\arctan(x) - \frac{1}{2}\ln(1+x^2) + \alpha = 0\,,$$

depending on the value of the parameter $\alpha \in \mathbb{R}$.
