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CALCULUS – EVALUATION TEST 12

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Problem 1. Consider the monotone increasing sequence $(a_n)_{n\in\mathbb{N}}$ defined by the *recursive* formula

$$a_0 = 1\,;$$
$$a_n = \sqrt{\frac{2+3a_{n-1}}{2}}\,, \quad \text{with } \ n \geq 1\,.$$

- Prove that the sequence is bounded.
- Calculate $\lim_{n\to\infty} a_n$.

Problem 2. Consider the function

$$f(x) \;=\; \begin{cases} \arctan\left(\dfrac{1}{x^2}\right) + \dfrac{\pi}{2}\,, & \text{if } \; x \neq 0\,, \\ \pi\,, & \text{if } \; x = 0\,. \end{cases}$$

- Prove that the function $f(x)$ is differentiable for all $x \in \mathbb{R}$.
- Find for which values of $x \in \mathbb{R}$ the function $f(x)$ is increasing.

Problem 3. Let $F(x) = \int_0^{x^3} \ln\left(t^{\frac{1}{3}} + \frac{1}{2}\right)\, dt$.

(a) Find and classify the local extrema of $F(x)$ for $x \in (0, 1)$.

(b) Use the Maclaurin polynomial of degree 3 for $F(x)$ to approximate $F(0.2)$.

Problem 4. Calculate $\int_e^5 \frac{dx}{x \ln(x)}$.

Problem 5. Study the convergence of the *improper* integral $\int_0^\infty \frac{|\sin(x)|}{x+x^2}\,dx$.
