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CALCULUS – EVALUATION TEST 13

Filippo Terragni, Eduardo Sánchez Villaseñor, Manuel Carretero Cerrajero

Problem 1. Prove that 5 is an upper bound for the *recursive* sequence defined as

$$a_1 = 0; \quad a_{n+1} = 4 + \frac{1}{5} a_n, \quad \text{for } n \geq 1.$$

Then, for $n \geq 1$, check that

$$a_n = 5 - \frac{1}{5^{n-2}}.$$

Problem 2. Determine the exact number of real solutions of the equation

$$e^{-x} - e^{x} - \ln(x) = 0\,, \quad \text{with} \;\; x \in (0, +\infty)\,.$$

Problem 3. Let

$$F(x) = \int_0^x e^{-t^2} dt\,.$$

Use a Taylor polynomial of degree 3 for $F(x)$ to approximate the value $F(1/10)$ and find an *upper bound* for the involved error.

Problem 4. Find *all* differentiable functions $F : (0, +\infty) \to \mathbb{R}$ that satisfy

$$F'(x) = \ln^2(x)\,, \quad F(1) = 0\,.$$

Problem 5. Find *all* values of $a, b \in \mathbb{R}$ that make the function

$$f(x) = \begin{cases} a + \displaystyle\int_0^{2x} \frac{\sin(t)}{t} dt\,, & \text{if } \quad x < 0\,, \\ \sqrt{2} + b\cos(2x)\ln(1+3x)\,, & \text{if } \quad x \geq 0\,, \end{cases}$$

continuous and differentiable in the domain.
