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CALCULUS – EVALUATION TEST 14

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Problem 1. Knowing that the *recursive* sequence defined as

$$a_1 = 0\,; \quad a_{n+1} = \sqrt{\frac{2+3\,a_n}{2}}, \quad \text{with } \; n \in \mathbb{N}\,,$$

is bounded, prove that it is increasing and calculate $\lim_{n\to\infty} a_n$.

Problem 2. Given $n \in \mathbb{N}$, consider

$$f(x) = \begin{cases} -\arctan(\ln(x^{2n}))\,, & \text{if} \quad x \neq 0\,, \\ \pi/2\,, & \text{if} \quad x = 0\,. \end{cases}$$

(a) Study the continuity and differentiability of $f(x)$.

(b) Find the intervals where $f(x)$ is increasing.

Problem 3. Let $n \in \mathbb{N}$. Then, calculate

$$\lim_{x \to 0} \frac{x \displaystyle\int_0^x e^{t^2}\,dt}{\displaystyle\int_0^x e^{t^2} \sin(t)\,dt}; \qquad \lim_{x \to 0} \frac{e^x - 1 - x - \frac{x^2}{2!} - \ldots - \frac{x^n}{n!}}{x^{n+1}}.$$

Problem 4. Let $G : \mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R}$ be defined by $G(x) = \displaystyle\int_{1/x}^{x^2+x} \frac{1}{2+t^2}\,dt$.

(a) Study the continuity and differentiability of $G(x)$.

(b) Find the function $G'(x)$.

(c) Calculate the indefinite integral

$$\int \frac{1}{2+t^2}\,dt$$

and use the result to determine the value of $\lim\limits_{x\to+\infty} G(x)$.

Problem 5. Calculate $\int \ln^2(x)\,dx$.
