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CALCULUS – EVALUATION TEST 15

Filippo Terragni, Eduardo Sánchez Villaseñor, Manuel Carretero Cerrajero

Problem 1. Study the convergence of the series

$$\sum_{n=1}^{\infty} a_n \,,$$

where $(a_n)_{n\in\mathbb{N}}$ is the *recursive* sequence defined as

$$a_1 = 1\,; \quad a_{n+1} = -\frac{a_n}{2}\left(1+\frac{1}{n}\right)^{n/2}, \quad \text{with } n \in \mathbb{N}\,.$$

Problem 2. Find the exact number of real solutions of the equation

$$e^x = ax\,,$$

depending on the value of $a \in \mathbb{R}$.

Problem 3. Consider the function

$$f(x) = (36 + x^3)^{-1/2}, \quad \text{with } x \neq -6^{2/3}.$$

(a) Write the Taylor polynomial of degree 6 about $a = 0$ for $f(x)$.

(b) Find a rational number that approximates $f(-1)$ within an error smaller than 10^{-2}.

Problem 4. Let $f, F : [0, 7] \longrightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 1, & \text{if } 0 \leq x \leq 4, \\ 5 - x, & \text{if } 4 < x \leq 5, \\ -1, & \text{if } 5 < x \leq 7, \end{cases} \qquad F(x) = \int_0^x f(t)\, dt.$$

(a) Calculate the values $F(4)$, $F(5)$, and $F(7)$.

(b) Study the continuity and differentiability of $F(x)$.

Problem 5. Calculate

$$\lim_{x \to 0} \frac{\sin(x)\cos(x) - \arctan(x)}{\ln(1 + x^3)}$$

by using appropriate Taylor polynomials.

Problem 6. Calculate $\int x \arctan(x)\, dx$.
