

OpenCourseWare

CALCULUS – EVALUATION TEST 1

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Problem 1. Consider the sequence $(a_n)_{n \in \mathbb{N}}$ of real numbers defined by

$$a_n = -4 + \frac{a_{n-1}}{3}, \quad \text{with } n \geq 2;$$
$$a_1 = 0.$$

- Prove that the sequence is decreasing.
 - Prove that the sequence is bounded.
 - Calculate $\lim_{n \rightarrow \infty} a_n$.
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Problem 2. Find *all* values of the parameter $\alpha \in \mathbb{R}$ such that the series

$$\sum_{n=0}^{\infty} \frac{(\alpha - 2)^n}{n^2 + 1}$$

is convergent.

Problem 3. Consider the function

$$f(x) = \begin{cases} -x^2 - 7 \cos\left(\frac{\pi}{2}x\right) & \text{if } x > 2, \\ a(x+1) + b & \text{if } -1 < x \leq 2, \\ x^3 - 12x + 5 & \text{if } x \leq -1, \end{cases}$$

where a and b are real numbers.

- Find the values of a and b such that $f(x)$ is continuous in \mathbb{R} .
 - Calculate (if any) the local maxima and minima of $f(x)$ for $x < -1$.
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Problem 4. Approximate the value

$$\ln\left(\frac{4}{3}\right)$$

by using a polynomial of degree 3 and find an appropriate *upper bound* for the involved error.
