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CALCULUS – EVALUATION TEST 2

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Problem 1. Consider the monotone increasing sequence $(a_n)_{n \in \mathbb{N}}$ defined by the following *recursive* formula

$$a_1 = 0 \, ;$$
$$a_{n+1} = \sqrt{4a_n + 5} \, , \quad \text{with } n \geq 1 \, .$$

Prove that the sequence is bounded and calculate $\lim_{n \to \infty} a_n$.

Problem 2. Find *all* values of the parameter $x \in \mathbb{R}$ such that the series

$$\sum_{k=1}^{\infty} \frac{3^{2k} x^{3k}}{(2k+1)\, 5^k}$$

is convergent.

Problem 3. Consider the function

$$F(x) = \int_0^{5x} e^{-7t^4} dt, \quad \text{with } x \in \mathbb{R}.$$

- Prove that $F(x)$ is *odd*.

- Prove the existence of the limit $\ell = \lim_{x \to \infty} F(x)$.

- Prove that the function $F : \mathbb{R} \to (-\ell, \ell)$ is monotone *increasing*.

- Calculate $(F^{-1})'(0)$.

- Calculate $\lim_{x \to 0} \dfrac{5x - F(x)}{x^5}$.

Problem 4. Calculate

$$\int \frac{dx}{(x+1)^{4/3} - (x+1)^{2/3}}.$$