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CALCULUS – EVALUATION TEST 3

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Problem 1. Find *all* values of the parameter $x \in \mathbb{R}$ such that the series

$$\sum_{k=1}^{\infty} \frac{\sin^k(x/3)}{k^{1/5} + k^{1/6}}$$

converges.

Problem 2. Consider the function $f : [-1, 1] \longrightarrow \mathbb{R}$ such that

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } -1 \leq x < 0\,, \\ x & \text{if } \quad 0 \leq x \leq 1\,. \end{cases}$$

- Prove that $f(x)$ is bounded and calculate its image.
- Study the differentiability of $f(x)$ in the interval $(-1, 1)$.

Problem 3.

- Approximate the value $\sqrt[7]{6/5}$ by using a polynomial of degree 2.
- Find a proper *upper bound* for the error involved in the previous approximation.

Problem 4. Calculate

$$\lim_{x\to 0}\left(\frac{1}{x^4}-\frac{1}{x^7}\int_0^x \sin(3t^2)\,dt\right).$$

Problem 5. Calculate the definite integral

$$\int_0^{\ln \sqrt{2}} \sqrt{e^{2t} - 1}\, dt\,.$$

Problem 6. Study the convergence of the family of *improper* integrals given by

$$I_n(\lambda) = \int_0^{+\infty} x^n e^{-\lambda x}\, dx \,, \quad \text{with} \;\; n = 0, 1, 2, \dots \,,$$

where $\lambda > 0$.
