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CALCULUS – EVALUATION TEST 4

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Problem 1. Consider the monotone decreasing sequence $(a_n)_{n \in \mathbb{N}}$ defined by the *recursive* formula

$$a_1 = 1 \, ;$$
$$a_n = -8 + \frac{a_{n-1}}{3} \, , \quad \text{with } n \geq 2 \, .$$

- Prove that the sequence is bounded.

- Calculate $\lim_{n \to \infty} a_n$.

Problem 2. Find *all* values of the parameter $\alpha \in \mathbb{R}$ such that the series

$$\sum_{k=1}^{\infty} (-1)^k \frac{3^k \alpha^{2k}}{k+1}$$

is convergent.

Problem 3. Approximate the value

$$\sqrt[3]{1.1}$$

by a polynomial of degree 2 and find an appropriate *upper bound* for the involved error.

Problem 4. Given the function
$$f(x) = x^x,$$
find the exact number of real solutions of the equation $f(x) = 2$ in the interval $[1, +\infty)$.