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CALCULUS – EVALUATION TEST 5

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Problem 1. Consider the monotone sequence $(a_n)_{n \in \mathbb{N}}$ defined by the *recursive* formula

$$a_1 = 0 \, ; \qquad a_n = \sqrt{a_{n-1} + 20} \, , \quad \text{with} \ \ n \geq 2 \, .$$

Prove that the sequence is bounded and calculate $\displaystyle \lim_{n \to \infty} a_n$.

Problem 2. Calculate

$$\lim_{x \to 0} \frac{\sqrt{1 + x^2} + 2x + x \arctan(x) - e^{3x}\left[1 - \ln(1 + x)\right]}{x\left[\ln(1 + 5x) + \arctan(2x)\right]}.$$

Problem 3. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \int_0^x e^{1-\sqrt{1+t^2}} \, dt \, .$$

- Prove that $f(x)$ is *odd*.
- Prove that $f(x)$ is *increasing*.
- Find the Taylor polynomial of degree 3 about $x_0 = 0$ for $f(x)$.
- Study the convergence of the *improper* integral

$$\lim_{x \to +\infty} f(x) = \int_0^\infty e^{1-\sqrt{1+t^2}} \, dt \, .$$

Problem 4. Calculate

$$\int \frac{\sin(x^{1/3})}{x^{1/3}}\, dx$$

in terms of elementary functions.